

# Table of Contents

---

Preface . . . . .	xiii
<b>Chapter 1 First Principles</b> . . . . .	1
1.1 A typical example . . . . .	1
1.2 Morphological convolution . . . . .	5
1.3 A bird's eye view . . . . .	10
1.4 Historical remarks . . . . .	15
<b>Chapter 2 Complete Lattices</b> . . . . .	17
2.1 Basic concepts . . . . .	17
2.2 Boolean lattices . . . . .	28
2.3 Regular closed sets . . . . .	34
2.4 Boolean functions . . . . .	37
2.5 Bibliographical notes . . . . .	43
<b>Chapter 3 Operators on Complete Lattices</b> . . . . .	44
3.1 Lattice operators . . . . .	45
3.2 Adjunctions . . . . .	49
3.3 Openings and closings . . . . .	55
3.4 Conditional operators . . . . .	61
3.5 Activity ordering . . . . .	63
3.6 The centre on non-Boolean lattices . . . . .	67
3.7 Bibliographical notes . . . . .	69

<b>Chapter 4 Operators which are Translation Invariant . . . . .</b>	71
4.1 Set model for binary images . . . . .	71
4.2 Hit-or-miss operator . . . . .	74
4.3 Dilation and erosion . . . . .	80
4.4 Opening and closing . . . . .	88
4.5 Boolean functions . . . . .	96
4.6 Grey-scale morphology . . . . .	102
4.7 Bibliographical notes . . . . .	117
<b>Chapter 5 Adjunctions, Dilations, and Erosions . . . . .</b>	122
5.1 General properties of adjunctions . . . . .	123
5.2 T-invariance: the abelian case . . . . .	126
5.3 Self-dual and Boolean lattices . . . . .	136
5.4 Representation theorems . . . . .	138
5.5 Translation invariant morphology . . . . .	142
A. The Boolean lattice $\mathcal{P}(\mathbb{E}^d)$ . . . . .	142
B. The closed sets $\mathcal{F}(\mathbb{R}^d)$ . . . . .	143
C. Convex subsets of $\mathbb{R}^d$ . . . . .	145
D. Matrix morphology . . . . .	146
5.6 Polar morphology . . . . .	147
5.7 Grey-scale functions . . . . .	150
A. Additive structuring functions . . . . .	150
B. Multiplicative structuring functions . . . . .	152
5.8 T-invariance: the nonabelian case . . . . .	154
A. Homogeneous spaces . . . . .	154
B. T-operators on $\mathcal{P}(T)$ . . . . .	157
C. Projection and lift operator . . . . .	159
D. T-operators on $\mathcal{P}(E)$ . . . . .	163
5.9 Translation-rotation morphology . . . . .	170
5.10 Bibliographical notes . . . . .	175
<b>Chapter 6 Openings and Closings . . . . .</b>	177
6.1 Algebraic theory of T-openings . . . . .	178
6.2 Self-dual and Boolean lattices . . . . .	187
6.3 Adjunctional openings and closings . . . . .	188
6.4 T-openings: the nonabelian case . . . . .	192
6.5 Annular openings . . . . .	195
6.6 Openings from inf-overfilters . . . . .	199

6.7	Granulometries . . . . .	207
6.8	Dominance and incidence structures . . . . .	211
6.9	Bibliographical notes . . . . .	215
<b>Chapter 7 Hit-or-Miss Topology and Semi-Continuity . . . . .</b>		218
7.1	Topology: basic concepts . . . . .	219
7.2	Metric spaces . . . . .	225
7.3	Hausdorff metric . . . . .	229
	A. Closed sets . . . . .	229
	B. Compact sets . . . . .	233
7.4	Hit-or-miss topology . . . . .	235
7.5	Myope topology . . . . .	241
7.6	Semi-continuity . . . . .	245
7.7	Basis representations . . . . .	251
7.8	Bibliographical notes . . . . .	252
<b>Chapter 8 Discretization . . . . .</b>		255
8.1	Statement of the problem . . . . .	255
8.2	Morphological sampling . . . . .	257
8.3	Discretization of images . . . . .	261
8.4	Discretization of operators . . . . .	263
8.5	Covering discretization . . . . .	265
8.6	Bibliographical notes . . . . .	270
<b>Chapter 9 Convexity, Distance, and Connectivity . . . . .</b>		272
9.1	Convexity . . . . .	273
9.2	Geodesic distance and M-convexity . . . . .	284
9.3	Metric dilations . . . . .	293
9.4	Distance transform . . . . .	297
9.5	Geodesic and conditional operators . . . . .	303
9.6	Granulometries . . . . .	308
9.7	Connectivity . . . . .	316
9.8	Skeleton . . . . .	319
9.9	Discrete metric spaces . . . . .	325
9.10	Bibliographical notes . . . . .	331
<b>Chapter 10 Lattice Representations of Functions . . . . .</b>		336
10.1	Introduction . . . . .	337

10.2	Admissible complete lattices . . . . .	339
10.3	Power-type lattices . . . . .	342
10.4	Function representations . . . . .	345
10.5	Semi-continuous functions . . . . .	347
10.6	Extension of lattice operators . . . . .	349
10.7	Lattices with negation . . . . .	353
10.8	Operators: from sets to functions . . . . .	356
10.9	Bibliographical notes . . . . .	359
<b>Chapter 11 Morphology for Grey-Scale Images . . . . .</b>		360
11.1	Functions and threshold sets . . . . .	361
11.2	Semi-flat function operators . . . . .	362
11.3	Flat function operators . . . . .	364
11.4	Flat operators and Boolean functions . . . . .	371
11.5	H-operators . . . . .	373
11.6	Umbra transform . . . . .	376
11.7	Grey-value set $\bar{Z}$ . . . . .	381
11.8	Finite grey-value sets . . . . .	382
11.9	Finite grey-value sets and truncation . . . . .	385
11.10	Geodesic and conditional operators . . . . .	392
11.11	Granulometries . . . . .	396
	A. (T,T)-Minkowski granulometries . . . . .	398
	B. (T,H)-Minkowski granulometries . . . . .	399
11.12	Bibliographical notes . . . . .	404
<b>Chapter 12 Morphological Filters . . . . .</b>		408
12.1	Filters, overfilters, etc. . . . .	408
12.2	Lattice of filters . . . . .	414
12.3	Lattice of strong filters . . . . .	420
12.4	Invariance domain . . . . .	423
12.5	The middle filter . . . . .	427
12.6	Alternating sequential filters . . . . .	430
12.7	Bibliographical notes . . . . .	435
<b>Chapter 13 Filtering and Iteration . . . . .</b>		436
13.1	Order convergence . . . . .	437
13.2	Order continuity . . . . .	439
13.3	Relation with the hit-or-miss topology . . . . .	443

13.4	Translation invariant set operators . . . . .	445
13.5	Finite window operators . . . . .	448
13.6	Iteration and idempotence . . . . .	451
13.7	Iteration of the centre operator . . . . .	457
13.8	From centre operator to middle filter . . . . .	464
13.9	Self-dual operators and filters . . . . .	466
13.10	Bibliographical notes . . . . .	477
	 Bibliography . . . . .	479
	 Notation Index . . . . .	493
	 Subject Index . . . . .	499