

# Contents

FOREWORD	xi
PREFACE	xii
ACKNOWLEDGMENTS	xiii
<b>0 Introduction</b>	
0-1 Historical Development and General Review	2
0-2 Fundamental Rules and Basic Equations	4
Reference	11
<b>1 The Electron Optical Imaging System and Its Aberrations</b>	
1-1 Fundamentals	14
1-1-1 Rotationally Symmetric Electrostatic and Magnetic Fields	14
1-1-2 Electron Trajectories in Static Electric and Magnetic Fields	19
1-1-3 Gaussian Dioptrics	24
1-1-4 Electron Lenses	35
1-2 Geometrical Aberrations of Rotationally Symmetric Electron Optical Systems	46
1-2-1 The Description and Calculation of Aberrations	46
1-2-2 The Geometrical Aberration Figures	57
1-3 Chromatic, Asymmetric and Diffraction Aberration	69
1-3-1 Chromatic Aberrations	69
1-3-2 Asymmetric Aberrations	74
1-3-3 Diffraction Aberrations	81
1-4 The Measurement and Correction of Aberrations	92
1-4-1 The Measurement of Aberrations	92
1-4-2 Correction of Aberrations	100
1-5 Asymptotic Aberrations of Rotationally Symmetric Lenses	101
1-5-1 The Description and Calculation of Asymptotic Aberrations	101
1-5-2 Transfer Properties of Aberrations in Multilens Systems	109
1-6 The Linear Transformation of Gaussian Trajectory Parameters and Its Influence upon Aberrations	117
References	122
<b>2 The Electromagnetic Deflection System and Its Aberrations</b>	
2-1 Fundamentals of the Magnetic Deflection System	126
2-1-1 Structure and Field Distribution of Magnetic Deflection Systems	126
2-1-2 The Variational Principle and Trajectory Equation	130
2-1-3 Gaussian Deflection Properties	132

2-2	Third Order Aberrations of Magnetic Deflection Systems	134
2-2-1	Third Order Aberration Formulae and Coefficients	134
2-2-2	Classification of the Third Order Aberrations and Their Figures	140
2-2-3	Measurements of Magnetic Deflection Aberrations	148
2-3	Fundamentals of the Electrostatic Deflection System and Its Aberrations	151
2-3-1	Structure and Field Distribution of Electrostatic Deflection Systems	151
2-3-2	The Variational Principle and Trajectory Equation	154
2-3-3	Classification of the Third Order Aberrations and Their Figures	156
2-4	The Combined System (Magnetic Round Lens with Magnetic Deflector) and Its Aberrations	158
2-4-1	General Expression for Magnetic Field and Variational Function	158
2-4-2	Gaussian Trajectory and the Third Order Aberration Equation	161
2-4-3	Third Order Aberrations Expressed by $F_{4C}$	167
2-4-4	Third Order Aberrations Expressed by $F_{4D}$	172
2-5	The Combined System (Electrostatic Round Lens with Magnetic Deflector) and Its Aberrations	175
2-5-1	The Electrostatic Field, Variational Function and Trajectory Equation	176
2-5-2	The Third Order Aberrations for Electrostatic Combined Systems	178
2-6	Applications of the Theory for the Combined System	181
2-6-1	The Aberrations of a Combined System with Pre-deflection	181
2-6-2	The Moving Objective Lens in the Scanning Electron Beam System	184
	References	186

### 3 The Electromagnetic Multipole System and Its Aberrations

3-1	General Expressions for Electrostatic and Magnetic Fields	188
3-1-1	The Series Expansion for the Electric Potential of Nonrotationally Symmetric Electric Fields	188
3-1-2	The Series Expansions for Magnetic Scalar and Vector Potentials of Nonrotationally Symmetric Magnetic Fields	193
3-2	Analysis and Calculation of Static Electromagnetic Multipole Fields	198
3-2-1	The General Expression for the Electric and Magnetic Potentials in Cylindrical Coordinates	198
3-2-2	The Analysis and Calculation of Electromagnetic Multipole Fields	202
3-3	Electron Optical Properties and Geometrical Aberrations for Electromagnetic Multipole Systems	205
3-3-1	The General Variational Function and Its Expansion	205
3-3-2	Gaussian Trajectory Equation and Gaussian Dioptries	210
3-3-3	The Aberration Equation and Real Aberration Coefficients (Spherical Aberration and Distortion)	216
3-3-4	The Aberration Patterns of Spherical Aberrations and Distortions	220
3-4	Asymptotic Aberrations of Quadrupole Lenses	224
3-5	Chromatic Aberration of Electromagnetic Multipole Systems	229
3-6	Applications of the Multipoles as an Aberration Corrector	233
3-6-1	Aberration Correction in Fixed Electron Beam Systems	235
3-6-2	Aberration Correction in Scanning Electron Beam Systems	239
3-7	Image Defects Caused by Imperfections in Manufacture and Alignment	242
3-7-1	The Scalar Potential Distribution and the First Order Trajectory Equation for Magnetic Quadrupoles	243
3-7-2	Aberration Equations and Aberration Coefficients	248
	References	250

## 4 The Ion Optical System and Its Aberrations

4-1	Fundamentals	254
4-1-1	Electrostatic and Magnetic Fields	254
4-1-2	The Principle of Variation and the Trajectory Equation	266
4-2	Ion Lenses and Gaussian Dioptrics	272
4-2-1	Gaussian Dioptrics of Sector Fields with Perpendicular Entrance and Exit	273
4-2-2	Gaussian Dioptrics of Sector Fields with Oblique Entrance and Exit	280
4-2-3	First Order Theory of Double Focusing Spectrometers	285
4-3	The Second Order Aberration Theory	294
4-3-1	The Second Order Trajectory and Aberration for Cylindrical Electric Fields	295
4-3-2	The Second Order Trajectory and Aberration for Toroidal Electric Fields	301
4-3-3	The Second Order Trajectory and Aberration for Homogeneous Magnetic Fields	306
4-3-4	The Second Order Trajectory and Aberration for Inhomogeneous Magnetic Fields	311
4-3-5	The Second Order Trajectory and Aberration for Crossed Toroidal Electric and Inhomogeneous Magnetic Fields	316
4-4	Generalization of the Second Order Aberration Theory	328
4-4-1	Method of Matrix Calculation in Ion Optics	328
4-4-2	The Oblique Entrance and Exit	333
4-4-3	Curved Field Boundaries	341
4-4-4	The Effect of Fringing Fields	345
4-4-5	Liouville Theorem and Its Consequences	356
4-5	Developments in Double Focusing Mass Spectrometers	359
	References	364

## 5 Computer Aided Design of Electron and Ion Optical Systems

5-1	Calculation of Electromagnetic Fields by the Finite Difference Method	368
5-1-1	Fundamentals and Formulae	368
5-1-2	Successive Overrelaxation Iteration	372
5-1-3	Calculation of Potential and Field Strength. Plotting of Equipotential	374
5-2	Calculation of Electromagnetic Multipole Fields by the Fourier Expansion Method	379
5-2-1	Fourier Expansions for Multipole Fields	379
5-2-2	The Configurations of Multipole Fields and Boundary Conditions	380
5-2-3	The Successive Overrelaxation Method for Multipole Fields	385
5-2-4	The Plotting of Equipotential and Flux	387
5-3	The Calculation of Electromagnetic Fields by the Finite Element Method	390
5-3-1	The Calculation of the Magnetic Scalar Potential for Unsaturated Magnetic Lenses	390
5-3-2	The Calculation of Field Distributions Throughout Entire Magnetic Circuits in Unsaturated Magnetic Lenses	396
5-3-3	The Calculation of Field Distributions Throughout Entire Magnetic Circuits in Saturated Magnetic Lenses	400

5-4	The Calculation of Charged Particle Trajectories	401
5-4-1	The Statement of the Problem	401
5-4-2	Runge-Kutta Single-Step Methods	404
5-4-3	Hamming Multistep Methods	408
5-5	The Calculation of Gaussian Optical Parameters and Aberration Coefficients	411
5-5-1	The Solution of Gaussian Trajectory Equations	411
5-5-2	The Calculation of Gaussian Optical Parameters	413
5-5-3	The Calculation of Aberration Coefficients	417
	References	421
	Appendix: Additional Recent Publications by the Author (1982-1985)	423