

Contents

	Foreword	xiii
	Preface	xvii
	Acknowledgments	xxi
Part 1	Primer	
	One Dimensional Electrostatic and Electromagnetic Codes	1
<hr/>		
	1 Why Attempting to Do Plasma Physics via Computer Simulation Using Particles Makes Good Physical Sense	3
	2 Overall View of a One Dimensional Electrostatic Program	7
	2-1 Introduction	7
	2-2 The Electrostatic Model: General Remarks	8
	2-3 The Computational Cycle: General Remarks	11
	2-4 Integration of the Equations of Motion	12
	2-5 Integration of the Field Equations	16
	2-6 Particle and Force Weighting; Connection between Grid and Particle Quantities	19
	2-7 Choice of Initial Values; General Remarks	22
	2-8 Choice of Diagnostics; General Remarks	25
	2-9 Are the Results Correct? Tests	26

3	A One Dimensional Electrostatic Program ES1	29
3-1	Introduction	29
3-2	General Structure of the Program, ES1	29
3-3	Data Input to ES1	33
3-4	Change of Input Parameters to Computer Quantities	34
3-5	Normalization; Computer Variables	35
3-6	INIT Subroutine; Calculation of Initial Charge Positions and Velocities	36
3-7	SETRHO, Initialization of Charge Density	39
3-8	FIELDS Subroutine; Solution for the Fields from the Densities; Field Energy	40
3-9	CPFT, RPFT2, RPFT12, Fast Fourier Transform Subroutines	43
3-10	SETV, Subroutine for Initial Half-Step in Velocity	43
3-11	ACCEL, Subroutine for Advancing the Velocity	44
3-12	MOVE, Subroutine for Advancing the Position	48
3-13	Advance Time One Step	50
3-14	HISTORY Subroutine; Plots versus Time	50
3-15	Plotting and Miscellaneous Subroutines	51
4	Introduction to the Numerical Methods Used	55
4-1	Introduction	55
4-2	Particle Mover Accuracy; Simple Harmonic Motion Test	55
4-3	Newton Lorentz Force; Three-Dimensional $\mathbf{v} \times \mathbf{B}$ Integrator	58
4-4	Implementation of the $\mathbf{v} \times \mathbf{B}$ Rotation	61
4-5	Application to One-Dimensional Programs	63
4-6	Particles as Seen By the Grid; Shape Factors $S(x)$, $S(k)$	65
4-7	A Warm Plasma of Finite-Size Particles	68
4-8	Interaction Force with Finite-size Particles in a Grid	70
4-9	Accuracy of the Poisson Solver	72
4-10	Field Energies and Kinetic Energies	73
4-11	Boundary Conditions for Charge, Current, Field, and Potential	75
5	Projects for ES1	81
5-1	Introduction	81
5-2	Relations among Initial Conditions; Small Amplitude Excitation	81
5-3	Cold Plasma (or Langmuir) Oscillations; Analysis	86
5-4	Cold Plasma Oscillations; Project	90
5-5	Hybrid Oscillations; Project	92
5-6	Two Stream Instability; Linear Analysis	94
5-7	Two Stream Instability; an Approximate Nonlinear Analysis	98
5-8	Two Stream Instability; Project	104
5-9	Two Stream Instability; Selected Results	105
5-10	Beam Plasma Instability; Linear Analysis	110
5-11	Beam-Plasma Instability; an Approximate Nonlinear Analysis	114
5-12	Beam Plasma Instability; Project	119
5-13	Beam Cyclotron Instability; Linear Analysis	122
5-14	Beam Cyclotron Instability; Project	123

5-15	Landau Damping	124
5-16	Magnetized Ring Velocity Distribution; Dory-Guest-Harris Instability; Linear Analysis	127
5-17	Magnetized Ring-Velocity Distribution; Project	130
5-18	Research Applications	131
6	A 1d Electromagnetic Program EM1	133
6-1	Introduction	133
6-2	The One Dimensional Model	133
6-3	One Dimensional Field Equations and Integration	134
6-4	Stability of the Method	137
6-5	The EM1 Code, for Periodic Systems	138
6-6	The EM1BND Code, for Bounded Systems; Loading for $f(\mathbf{x}, \mathbf{v})$	139
6-7	EM1BND Boundary Conditions	141
6-8	EM1, EM1BND Output Diagnostics	142
7	Projects for EM1	145
7-1	Introduction	145
7-2	Beat Heating of Plasma	146
7-3	Observation of Precursor	149

Part 2 Theory

Plasma Simulation Using Particles in Spatial Grids with Finite Time Steps—Warm Plasma 153

8	Effects of the Spatial Grid	155
8-1	Introduction; Early Use of Grids and Cells with Plasmas	155
8-2	Spatial Grid Theory, Introduction	158
8-3	Some General Remarks on the Effects of a Periodic Spatial Nonuniformity	158
8-4	Notation and Conventions	161
8-5	Particle to Grid Weighting; Shape Factors	162
8-6	Momentum Conservation for the Overall System	164
8-7	Fourier Transforms for Dependent Variables; Aliasing due to Finite Fourier Series	165
8-8	More Accurate Algorithms Using Splines for $S(x)$	168
8-9	Generalization to Two and Three Dimensions	170
8-10	Linear Wave Dispersion	171
8-11	Application to Cold Drifting Plasma; Oscillation Frequencies	172
8-12	Cold Beam Nonphysical Instability	175
8-13	Solution for Thermal (Maxwellian) Plasma; Nonphysical Instabilities Caused by the Grid	177

9	Effects of the Finite Time Step	183			
9-1	Introduction	183			
9-2	Warm Unmagnetized Plasma Dispersion Function; Leapfrog Algorithm	184			
9-3	Alternative Analysis by Summation over Particle Orbit	191			
9-4	Numerical Instability	194			
9-5	The Dispersion Function Including Both Finite Δx and Δt	196			
9-6	Warm Magnetized Plasma Dispersion and Nonphysical Instability	197			
	a Derivation of the Dispersion Function	198			
	b Properties of the Dispersion Relation	200			
	c Numerical Instability	201			
9-7	Simulation of Slowly-Evolving Phenomena; Subcycling, Orbit-Averaging, and Implicit Methods	204			
	a Subcycling	201			
	b Implicit Time Integration	205			
	c Orbit Averaging	206			
9-8	Other Algorithms for Unmagnetized Plasma	206			
	a Class C Algorithms	207			
	b Class D Algorithms	210			
10	Energy-Conserving Simulation Models	213			
10-1	Introduction	213			
10-2	Nonexistence of a Conserved Energy in Momentum Conserving Codes	213			
10-3	An Energy-Conserving Algorithm	215			
10-4	Energy Conservation	218			
10-5	Algorithms Derived via Variational Principles	220			
10-6	Spatial Fourier Transforms of Dependent Variables	222			
10-7	Lewis's Poisson Difference Equation and the Coulomb Fields	222			
10-8	Small-Amplitude Oscillations of a Cold Plasma	223			
10-9	Lack of Momentum Conservation	225			
10-10	Aliasing and the Dispersion Relation for Warm Plasma Oscillations	228			
10-11	The Linear-Interpolation-Model Example	229			
	a Momentum Conservation and Self-Forces	229			
	b Macroscopic Field Accuracy	231			
10-12	The Quadratic Spline Model	232			
11	Multipole Models	235			
11-1	Introduction	235			
11-2	The Multipole Expansion Method	236			
11-3	The "Subtracted" Multipole Expansion	240			
11-4	Multipole Interpretations of Other Algorithms	242			
11-5	Relations between Fourier Transforms of Particle and Grid Quantities	245			
11-6	Overall Accuracy of the Force Calculation; Dispersion Relation	248			
11-7	Summary and a Perspective	251			
12	Kinetic Theory for Fluctuations and Noise; Collisions	255			
12-1	Introduction	255			
12-2	Test Charge and Debye Shielding	257			
12-3	Fluctuations	259			
	a The Spectrum	259			
	b Limiting Cases	261			
	1 Fluctuation-Dissipation Theorem	261			
	2 Spatial Spectrum	261			
	3 $\Delta x \neq 0, \Delta t = 0$ High-Frequency noise	263			
	4 $\Delta x = 0, \Delta t \neq 0$	263			
	5 $\Delta x, \Delta t$ Both Nonzero	264			
12-4	Remarks on the Shielding and Fluctuation Results	265			
12-5	Derivation of the Kinetic Equation	266			
	a Velocity Diffusion	266			
	b Velocity Drag	268			
	c The Kinetic Equation	269			
12-6	Exact Properties of the Kinetic Equation	271			
12-7	Remarks On the Kinetic Equation	274			
13	Kinetic Properties: Theory, Experience, and Heuristic Estimates	277			
13-1	Introduction	277			
13-2	The One-Dimensional Plasma in Thermal Equilibrium	277			
	a The Sheet Model	277			
	b The Equilibrium Velocity Distribution Is Maxwellian	279			
	c Debye Shielding	280			
	d Velocity Drag	282			
	e Relaxation Time	285			
13-3	Thermalization of a One-Dimensional Plasma	286			
	a Fast Time-Scale Evolution	287			
	b Slow Time-Scale Evolution	288			
	c Effects of Space and Time Aliasing	292			
13-4	Numerical Heating or Cooling	293			
	a Self Heating in One Dimension	293			
	b Cooling Due to Damping in the Particle Equations of Motion	293			
	c Heuristic Estimates	295			
13-5	Collision and Heating Times for Two-Dimensional Thermal Plasma	295			
13-6	Unstable Plasma	300			

Part 3 Practice

Programs in Two and Three dimensions:
Design Considerations 303

14 Electrostatic Programs in Two and Three Dimensions 305

14-1	Introduction	305
14-2	An Overall 2d Electrostatic Program	308
14-3	Poisson's Equation Solutions	310
14-4	Weighting and Effective Particle Shapes in Rectangular Coordinates: $S(\mathbf{x})$, $S(\mathbf{k})$, Force Anisotropy	311
14-5	Doubly Periodic Model and Boundary Conditions	315
a	Doubly Periodic Poisson Solver	316
b	Periodic Boundary Conditions; $\mathbf{k} = 0$ Fields	317
14-6	Poisson's Equation Solutions for Systems Bounded in x and Periodic in y	318
14-7	A Periodic-Open Model Using Inversion Symmetry	322
14-8	Accuracy of Finite-Differenced Poisson's Equation	325
14-9	Accuracy of Finite-Differenced Gradient Operator	327
14-10	Poisson's Equation Finite-Differenced in Cylindrical Coordinates r , $r-z$, $r-\theta$	331
a	r only	332
b	$r-z$	333
c	$r-\theta$	336
14-11	Weighting in Cylindrical Coordinates for Particles and Fields	336
14-12	Position Advance for Cylindrical Coordinates	338
14-13	Implicit Method for Large Time Steps	339
a	Implicit Time Differencing of the Particle Equations of Motion	340
b	Direct Method with Electrostatic Fields; Solution of the Implicit Equations	341
c	A One-Dimensional Realization	342
d	General Electrostatic Case	344
14-14	Diagnostics	345
14-15	Representative Applications	348
a	Diffusion Across \mathbf{B}	349
b	Instabilities	349
c	Heating	350

15 Electromagnetic Programs in Two and Three Dimensions 351

15-1	Introduction	351
15-2	Time Integration of the Fields and Location of the Spatial Grids	352
15-3	Accuracy and Stability of the Time Integration	354
15-4	Time Integration of the Particle Equations	356
15-5	Coupling of Particle and Field Integrations	358

15-6	The $\nabla \cdot \mathbf{B}$ and $\nabla \cdot \mathbf{E}$ Equations: Ensuring Conservation of Charge	359
15-7	$\mathbf{A}-\phi$ Formulation	361
15-8	Noise Properties of Various Current Weighting Methods	362
15-9	Schemes for $\Delta t_{\text{particles}} > \Delta t_{\text{fields}}$	364
a	Subcycling of the Maxwell Equations	364
b	Fourier-Transform Field Integration	365
15-10	Periodic Boundary Conditions	367
15-11	Open Sided Boundary Conditions	367
a	The Longitudinal Field	367
b	Absorbing Outgoing Electromagnetic Waves in a Dissipative Region	368
c	A Simple Closure of the Maxwell Equations at the Open Boundaries	369
d	Boundary Conditions for Waves Incident at (almost) Any Angle	371
e	Particle Boundary Conditions	373
15-12	Conducting-Wall Boundary Conditions	373
a	Closure of Maxwell's Equations at the Walls	373
b	Electrostatic Solutions in 2d	375
c	Combined Particle and Field Calculation	376
15-13	Integrating Maxwell's Equations in Cylindrical Coordinates	377
15-14	Darwin, or Magnetoinductive, Approximation	379
15-15	Hybrid Particle/Fluid Codes	380
15-16	Implicit Electromagnetic Codes	381
15-17	Diagnostics	381
a	Particles	381
b	Fields	382
c	Histories	382
d	Remarks	382
15-18	Representative Applications	383
a	Interaction of Intense Laser Light with Plasma	383
b	Reversed-Field Configurations; Pinches	385
15-19	Remarks on Large-Scale Plasma Simulation	385

16 Particle Loading, Injection; Boundary Conditions and External Circuit 387

16-1	Introduction	387
16-2	Loading Nonuniform Distributions, $f_0(\mathbf{v})$ and $n_0(\mathbf{x})$; Inversion of Cumulative Distribution Function	388
16-3	Loading a Cold Plasma or Cold Beam	389
16-4	Loading a Maxwellian Velocity Distribution	390
16-5	Quiet Starts: Smooth Loading in $\mathbf{x}-\mathbf{v}$ Space; Use of Mixed-Radix Digit-Reversed Number Sets	393
16-6	Quiet Start: Multiple-Beam and Ring Instabilities and Saturation; Recurrences	394
16-7	Loading a Magnetized Plasma with a Given Guiding Center Spatial Distribution $n_0(\mathbf{x}_{gc})$	402

xii CONTENTS

16-8	Particle Injection and Absorption at Boundaries; Field Emission, Ionization, and Charge Exchange	405
16-9	Particle and Field Boundary Conditions for Axially Bounded Systems; Plasma Devices	408
	a Charge and Field Boundary Conditions in 1d	409
	b Solutions with an External Circuit	411
Part 4	Appendices	417
<hr/>		
A	Fast Fourier Transform Subroutines	419
	a Complex Periodic Discrete Fourier Transform	419
	b Transform of Real Valued Sequences, Two at a Time	421
	c Sine Transform of Real-Valued Sequences, Two at a Time	422
	d Listings for CPFT, RPFT2, and RPFTI2	424
B	Compensating and Attenuating Functions Used in ES1	431
C	Digital Filtering in 1d and 2d	437
D	Direct Finite Difference Equation Solutions	443
E	Differencing Operators; Local and Nonlocal ($\nabla \rightarrow ik, \nabla^2 \rightarrow k^2$)	447
	References	453
	Author Index	465
	Subject Index	469