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# **Some Consequences of Unitarity and Crossing Existence and Asymptotic Theorems**

**D. ATKINSON**

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