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QUADRATIC TRANSPORT AND SOLUBLE BOLTZMANN EQUATIONS

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THE STATISTICAL MECHANICS OF THE ELECTRICAL DOUBLE LAYER

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Here the boundaries meet and all
Contradictions exist side by side

Dostoyevsky, *The Brothers Karamazov*

KERR EFFECT RELAXATION IN HIGH ELECTRIC FIELDS

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THE INTERNAL FIELD PROBLEM IN DEPOLARIZED LIGHT SCATTERING

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A CONSISTENT MOLECULAR TREATMENT OF DIELECTRIC PHENOMENA*

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