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# ATOMIC BETHE-GOLDSTONE EQUATIONS

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# ON THE USE OF THE CLUSTER EXPANSION AND THE TECHNIQUE OF DIAGRAMS IN CALCULATIONS OF CORRELATION EFFECTS IN ATOMS AND MOLECULES

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# THE CORRELATION ENERGY OF A NON-UNIFORM ELECTRON GAS

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# ELECTRON CORRELATION IN ATOMS AND MOLECULES

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# SOME ASPECTS ON THE CORRELATION PROBLEM AND POSSIBLE EXTENSIONS OF THE INDEPENDENT-PARTICLE MODEL\*

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# CORRELATION EFFECTS IN DIATOMIC MOLECULES OBTAINED FROM CONFIGURATION INTERACTION USING HARTREE-FOCK ORBITAL. EFFECTS ON ENERGY AND MONOELECTRONIC OPERATORS

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# A LINKED DIAGRAM TREATMENT OF CONFIGURATION INTERACTION IN OPEN- SHELL ATOMS

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# THE FIELD-THEORETIC FORM OF THE PERTURBATION THEORY FOR MANY-ELECTRON ATOMS

## I. ABSTRACT THEORY

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