# CONTENTS

Chapt	er Preface	Page vii
	Symbols	xiii
Ι	DEVELOPMENT OF THE EQUATIONS1.1Introduction1.2Properties of the electrically conducting fluid1.3Magnetohydrodynamic equations1.4Formulas for signal voltage1.5Selection of an instrument	1 1 4 7 11
II	MAGNETIC FIELD BALANCE2.1 Introduction2.2 Balance for instruments with DC primary field2.3 Balance for instruments with AC primary field	13 13 14
III	TYPICAL IONIZED GAS FLOWS AND INSTRUMENTATION         APPLICATIONS         3.1 Introduction       .       .       .         3.2 Discussion of plasma sheath       .       .       .         3.2.1 Parameters influencing the plasma sheath       .       .       .         3.2.2 Parameters characterizing the plasma sheath       .       .       .         3.3 Magnetohydrodynamic power generation       .       .       .         3.4 Wakes of projectiles at hypersonic speeds       .       .       .         3.5 Plasma of rocket exhausts       .       .       .       .         3.6 Ionization in shock tubes       .       .       .       .       .         3.7 Arc plasma generators       .       .       .       .       .       .	$     \begin{array}{r}       17 \\       17 \\       18 \\       20 \\       21 \\       22 \\       23 \\       23 \\       23 \\     \end{array} $
IV	PLASMA PROPERTIES MEASURED BY INDUCTIVE FLOW INSTRUMENTS4.1 Introduction4.1 Introduction4.2 Average $\sigma U$ transducers4.3 Electrical conductivity/velocity profile instruments4.4 Flow angle indicator4.5 Average electron collision frequency4.6 Velocity measurement and turbulence indicatorix	. 28 28 28 29 . 30 . 31

## CONTENTS

Chapte	er		Page		
V AVERAGE AND PROFILE $\sigma U$ TRANSDUCERS					
	5.1	Measuring the influence function	34		
	5.2	Average $\sigma U$ transducers	35		
		5.2.1 Additional design features	35		
		5.2.2 Example of an average $\sigma U$ transducer	37		
	5.3	Electrical conductivity/velocity profile transducer .	38		
		5.3.1 Requirements on the influence function	38		
		5.3.2 Vector analogy	40		
	5.4	Three different coil geometries for $\sigma U$ profile transducers	42		
		5.4.1 E-Lamination transducer with multiple sensing	~-		
		coils	42		
		5.4.2 Three-signal concentric coil arrangement	44		
		5.4.3 Nine-signal coil arrangement	44		
		5.4.4 Results of a test with the three-signal transducer.	48		
VI	TRA	NSDUCERS FOR ROCKET EXHAUSTS, ARC PLASMA JETS AND			
	Axis	YMMETRIC REENTRY VEHICLES	-		
	6.1	Description of the transducer	50		
	6.2	Influence function in cylindrical coordinates	51		
	6.3	Interpretation of the radial average	53		
	6.4	Axial gradients.	54		
	6.5	Calibration procedure for internal flow	57		
		6.5.1 Measurement of calibration constant	57		
		6.5.2 Measurement of influence function	58		
		6.5.3 Eddy current experiment	60		
	6.6	Calibration procedure for external flow	61		
VII	TRA	NSDUCERS FOR TRANSIENT FLOWS. SHOCK TUBES AND			
, 11	BAL	LISTIC RANGES			
	7.1	Introduction	65		
	7.2	Theory	65		
		7.2.1 Calibration using a conducting rod	67		
		7.2.2 Integration circuit	69		
		7.2.3 Calibration using a thin electrically conducting	r		
		disc	70		
	7.3	Design features.	72		
	7.4	Conductivity measurements at $R_{\rm m}$ greater than unity	74		
		<i>m</i> 3 <i>y y y y y y y y y y</i>			
VIII	FLO	W ANGLE INDICATOR			
	8.1	Introduction	77		
	8.2	Dependence of the signal on flow angle relative to the	5		
		transducer	. 77		
	8.3	Description of transducers and test apparatus	. 81		
	8.4	Experimental results	. 81		
		8.4.1 E-Lamination transducer	. 81		
		8.4.2 Pancake coil geometry	. 83		
	8.5	Zero angle of attack sensor	. 84		

# CONTENTS

Chapt	er	Page
IX	Average Electron Collision Frequency Instrument	0
	9.1 Introduction	87
	9.2 Electrical conductivity tensor	87
	9.3 Voltage induced in the sensing coil	89
	9.3.1 Signal from E-Lamination transducer	90
	9.3.2 Signal from pancake-coil transducer	91
	9.3.3 Signal for flow normal to reference axis of trans-	•-
	ducer	92
	9.4 Measurement of collision frequency	92
	9.5 Summary	93
		50
Х	TURBULENCE AND VELOCITY	
	10.1 Introduction	94
	10.2 Theoretical background	95
	10.2.1 Formula for the sensing coil voltage	95
	10.2.2 Estimate of signal magnitude	95
	10.3 Apparatus and typical results	95
	10.4 Discussion of results	99
	10.5 Turbulence indicator	101
		101
XI	STEADY MAGNETIC FIELD INSTRUMENTS FOR STEADY FLOWS	
	11.1 Introduction	105
	11.2 Detectors	105
	11.2.1 Fluxgate magnetometers	105
	11.2.2 Hall effect detector	107
	11.2.3 Comparison of fluxgate and Hall detector for	
	flight applications	107
	11.3 Relation of signal to plasma properties	108
	11.4 Design features.	109
	11.4.1 Transducer geometry and circuit diagram .	109
	11.4.2 Refinements to the instrument	111
	11.5 Design for MHD generators and accelerators	113
	11.6 Summary	114
XII	Other Diagnostic Instruments	
	12.1 Introduction	115
	12.2 Interlocked toroids for closed systems	115
	12.3 Radio frequency transducers	116
	12.3.1 Relevant parameters	116
	12.3.2 Analysis of infinite solenoid	116
	12.3.3 Analysis of single loop	121
	12.3.4 Radio frequency bridges	126
	12.3.5 Phase angle and impedance change: a com-	
	bination electrical and velocity transducer	130
	12.3.6 Solenoid with sensing coil	131
	12.3.7 Conductivity from change in phase angle	139
	12.5.7 Conductivity nom change in phase angle .	104

#### CONTENTS

Chapter		Page
•	12.3.8 Impedance change due to plasma	134
	12.3.9 Conductivity measurement from power dissipa-	
	tion	137
	12.3.10 Change in resonant frequency	142
	12.3.11 Relation of $Q$ to electrical conductivity	144
	12.3.12 Summary of RF methods	147
12.4	Langmuir probes	147
	12.4.1 Conventional Probe Theory	147
	12.4.2 Probe Theory for Moving Plasma	152
	12.4.3 Modified Langmuir Probes	154
12.5	Electromagnetic flow meters with electrodes	155
12.6	Measurement of electrical conductivity using electrodes	155
12.7	Microwaves as a diagnostic tool	155

## Appendices

A	Description of MHD Profile Meter and Transition Indicator for a Reentry Vehicle					
	A.1	Design guidelines				163
	A.2	Block diagram of instrument				165
	A.3	Transducer assembly .				168
	A.4	Performance of instrument				168

# B Design Features of Axial Flow Transducer for Rocket Exhaust

Author Index	175
Subject Index	179
TABLE OF CONSTANTS AND CONVERSION FACTORS	inside front cover
TABLE OF MAGNETOHYDRODYNAMICS AND PLASMA	inside
Formulae	back cover