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## TOKAMAK MHD INSTABILITIES

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## COLLECTIVE PROCESSES IN A TOKAMAK

## WITH HIGH-ENERGY PARTICLES

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## NONEIGENMODE INSTABILITIES

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INSTABILITIES OF QUASIODD PERTURBATIONS  
IN TOKAMAKS

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