

Contents

	<i>Preface</i>	xiii
1	Introduction	1
2	Basic properties of magnetohydrodynamics	8
2.1	The MHD equations	8
2.2	Conservation laws in ideal MHD	11
2.3	Magnetic helicity	14
2.4	Reduced MHD equations	17
2.5	Validity limits of ideal MHD and dissipation effects	21
3	Magnetostatic equilibria	24
3.1	One-dimensional configurations	24
3.2	The two-dimensional equilibrium equation	27
3.3	Some exact two-dimensional equilibrium solutions	32
3.3.1	Solov'ev's solution	32
3.3.2	Elliptical plasma cylinder	35
3.3.3	Constant- μ , force-free equilibria	38
3.3.4	Nonlinear equilibria with $j_z = e^{-\psi}$	40
3.4	Numerical solution of the Grad-Shafranov equation	42
3.5	Three-dimensional equilibria	44
3.5.1	The general equilibrium problem	44
3.5.2	Numerical equilibrium computations	47
4	Normal modes and instability	49
4.1	The normal mode problem in MHD	50
4.2	Waves in a homogeneous plasma	51
4.3	The energy principle	53
4.3.1	Proof of the energy principle	54
4.3.2	Different forms of the energy integral	56
4.4	The cylindrical pinch	60
4.4.1	Normal modes in a cylindrical pinch	60

4.4.2	The energy principle for a cylindrical equilibrium	62
4.5	The circular cylindrical tokamak	64
4.6	Toroidal effects on ideal tokamak stability	67
4.6.1	Interchange and ballooning modes	67
4.6.2	The toroidal internal kink mode	69
4.7	Resistive instabilities	71
4.7.1	The tearing mode	73
4.7.2	The resistive internal kink mode	77
4.7.3	Resistive energy principle	81
4.7.4	The toroidal tearing mode	83
5	Nonlinear evolution of MHD instabilities	85
5.1	The quasi-linear approximation	87
5.2	Nonlinear external kink modes	90
5.2.1	Nonlinear energy integral for free boundary modes	91
5.2.2	Vacuum bubbles	95
5.2.3	Effect of magnetic shear	101
5.3	Nonlinear theory of the ideal internal kink mode	103
5.4	The small-amplitude nonlinear behavior of the tearing mode	107
5.4.1	Standard low- β case	107
5.4.2	Influence of finite β and field line curvature	114
5.5	Saturation of the tearing mode	117
5.5.1	Quasi-linear theory of tearing-mode saturation	118
5.5.2	Influence of the equilibrium current profile	120
5.5.3	Effect of dynamic resistivity	122
6	Magnetic reconnection	127
6.1	Current sheets: basic properties	128
6.1.1	Sweet-Parker current sheet model	128
6.1.2	Effects of hyperresistivity and viscosity	131
6.2	Quasi-ideal models of stationary reconnection	132
6.2.1	Driven reconnection	132
6.2.2	Petschek's slow shock model	133
6.2.3	Syrovatskii's current sheet solution	138
6.3	Scaling laws in stationary current sheet reconnection	142
6.4	Current sheets: refined theory	147
6.4.1	Stationary solution in the vicinity of the neutral point	147
6.4.2	Current sheet edge region	150
6.5	Tearing instability of a Sweet-Parker current sheet	152
6.6	Examples of 2-D reconnecting systems	156
6.6.1	Coalescence of magnetic islands	156
6.6.2	Nonlinear evolution of the resistive kink mode	159
6.6.3	Plasmoids	164
6.7	Magnetic reconnection in general three-dimensional systems	166
6.8	Turbulent reconnection	172

7	MHD turbulence	175
7.1	Homogeneous isotropic turbulence	177
7.2	Properties of nondissipative MHD turbulence	179
7.2.1	Ideal invariants	179
7.2.2	Absolute equilibrium distributions	181
7.2.3	Cascade directions	183
7.3	Self-organization and turbulence decay laws	184
7.3.1	Selective decay	185
7.3.2	The α -term and turbulent dynamo theory	188
7.3.3	Dynamic alignment of velocity and magnetic field	193
7.3.4	Energy decay laws	195
7.4	Energy spectra	196
7.4.1	Inertial range spectra in MHD turbulence	196
7.4.2	Dissipation scales	200
7.4.3	Energy spectra in highly aligned systems	205
7.5	Closure theory for MHD turbulence	208
7.5.1	The problem of closure	209
7.5.2	Eddy-damped quasi-normal Markovian approximation	212
7.6	Energy dissipation in 2-D MHD turbulence	214
7.6.1	Spontaneous excitation of small-scale turbulence	214
7.6.2	Energy dissipation rates	216
7.7	Intermittency	217
7.7.1	The log-normal theory	218
7.7.2	The β -model and its generalizations	222
7.7.3	Probability distribution functions	225
7.7.4	Intermittency in MHD turbulence	228
7.8	Turbulent convection of magnetic fields	232
7.8.1	Magnetoconvection in 3-D systems	232
7.8.2	Convection of magnetic flux in 2-D	235
8	Disruptive processes in tokamak plasmas	239
8.1	Sawtooth oscillations	240
8.1.1	Early experimental observations	240
8.1.2	Kadomtsev's theory of the sawtooth collapse	242
8.1.3	Sawtooth behavior in large-diameter, high-temperature tokamak plasmas	247
8.1.4	Numerical simulations of sawtooth oscillations	249
8.1.5	Alternative concepts of fast sawtooth reconnection	253
8.1.6	Kink-mode stabilization and the problem of sudden collapse onset	255
8.1.7	Experimental observation of partial reconnection in the sawtooth collapse	257
8.1.8	Collapse dynamics at high S -values	260
8.2	Major disruptions	262
8.2.1	Disruption-imposed operational limits	262
8.2.2	Disruption dynamics	265

8.2.3	Single-helicity models	271
8.2.4	Hollow current profile disruptions	273
8.2.5	Multi-helicity dynamics	275
8.3	Edge-localized modes	281
8.3.1	The $L \rightarrow H$ transition	281
8.3.2	The ELM phenomenon	284
9	Dynamics of the reversed-field pinch	289
9.1	Minimum-energy states	290
9.1.1	Taylor's theory	290
9.1.2	Experimental results	294
9.2	Stability properties of RFP equilibria	297
9.2.1	Ideal instabilities	297
9.2.2	Resistive kink modes in the RFP	300
9.3	Single-helicity behavior	301
9.3.1	Nonlinear evolution of the $m = 1$ instability	301
9.3.2	Helical ohmic states	304
9.3.3	Numerical simulation of helical states	307
9.4	Three-dimensional RFP dynamics	309
9.4.1	Turbulent RFP dynamo	309
9.4.2	Quasi-linear theory of RFP dynamics	312
10	Solar flares	316
10.1	The solar convection zone	317
10.1.1	Phenomenological description of thermal convection	319
10.1.2	Compressible convection in a strongly stratified fluid	322
10.1.3	Solar magnetoconvection	324
10.2	Magnetic fields in the solar atmosphere	326
10.2.1	Structure of the solar atmosphere	326
10.2.2	Active regions	328
10.2.3	Magnetic buoyancy	329
10.2.4	Magnetic structures in active regions	330
10.3	Solar flares	331
10.3.1	Phenomenology of flares	332
10.3.2	Energy storage	335
10.3.3	Stability of twisted flux tubes	338
10.3.4	Current sheet formation and catastrophe theory in a sheared arcade	341
10.3.5	Dynamical models of plasmoid generation and eruption	345
	<i>Outlook</i>	349
	<i>References</i>	351
	<i>Index</i>	372