## Table of Contents

Pre	face to	o the English Edition	V		
Preface to the Russian Edition VII			VII		
Part I.	Finit	e-Dimensional Differential Geometry and Mechani	cs		
Chapte					
		etric Constructions in Calculus on Manifolds	3		
1.		plete Riemannian Metrics and the Completeness of	_		
	NAME OF STREET	or Fields	3		
	1.A		0		
	- T	the Completeness of a Vector Field			
Δ.	1.B	INTERNATION OF THE SUPPLEMENTAL SERVICES OF THE SUPPLEMENT OF THE SUPPLEMENTAL SERVICES OF THE SUPPLEMENT SERVICES OF THE SUPPLEMENTAL SERVICES OF THE SUPPLEMENTAL SERVICES OF THE SUPPLEMENT SERVICES OF THE SUPPLEM	Э		
2.		annian Manifolds	7		
9		essing a Uniform Riemannian Atlas			
3.		ral Operators with Parallel Translation	100 march		
	3.A 3.B	The Operator $S$	KCX-5-KK		
	3.C	Integral Operators			
Chapte	r 2				
Geome	tric Fe	ormalism of Newtonian Mechanics	17		
4.	Geon	netric Mechanics: Introduction and			
	Revie	ew of Standard Examples	17		
	4.A	Basic Notions			
	4.B	Some Special Classes of Force Fields			
	4.C	Mechanical Systems on Groups			
<b>5</b> .	Geon	netric Mechanics with Linear Constraints	22		
	5.A	Linear Mechanical Constraints	22		
	5.B	Reduced Connections	23		
	5.C	Length Minimizing and Least-Constrained			
		Nonholonomic Geodesics	24		
6.	Mechanical Systems with Discontinuous Forces and Systems				
	with	Control: Differential Inclusions	26		
7.	Integral Equations of Geometric Mechanics:				
		Velocity Hodograph			
	7.A	General Constructions	29		

	7.B Integral Formalism of Geometric Mechanics with	
	Constraints	31
8.	Mechanical Interpretation of Parallel Translation and	9-2-3-3
	Systems with Delayed Control Force	32
Chapte	r 3	
Accessi	ible Points of Mechanical Systems	39
9.	Examples of Points that Cannot Be Connected by	
	a Trajectory	40
10.		41
11.	Generalizations to Systems with Constraints	45
Part II	. Stochastic Differential Geometry and its Applications	
	Physics	
Chanta		
Chapte	stic Differential Equations	
	mannian Manifolds	40
		49
12.		40
	on Finite-Dimensional Linear Spaces	
	12.A Wiener Processes	
	12.B The Itô Integral	
	12.C The Backward Integral and the Stratonovich Integral 12.D The Itô and Stratonovich Stochastic Differential	53
	Equations	. 54
	12.E Solutions of SDEs	. 56
	12.F Approximation by Solutions of Ordinary Differential	
	Equations	. 57
	12.G A Relationship Between SDEs and PDEs	. 58
13.	Stochastic Differential Equations on Manifolds	. 59
14.	Stochastic Parallel Translation and the Integral Formalism	
	for the Itô Equations	67
15.		
	Stochastic Differential Equations	. 76
	15.A Wiener Processes on Riemannian Manifolds	. 76
	15.B Stochastic Equations	. 78
	15.C Equations with Identity as the Diffusion Coefficient .	. 80
16.	Stochastic Differential Equations with Constraints	. 83
Chapte	r 5	
· ·	ngevin Equation	. 87
	The Langevin Equation of Geometric Mechanics	
18.	Strong Solutions of the Langevin Equation,	
Parenter Carl	Ornstein-Uhlenbeck Processes	. 91

Chapte		
Mean I	erivatives, Nelson's Stochastic Mechanics, and	
Quantiz	ation	95
19.	More on Stochastic Equations and	
	Stochastic Mechanics in $\mathbb{R}^n$	96
	19.A Preliminaries	96
	19.B Forward Mean Derivatives	97
	19.C Backward Mean Derivatives and Backward	W-2007 <u>2</u> 6
	Equations	98
	19.D Symmetric and Antisymmetric Derivatives	101
	19.E The Derivatives of a Vector Field Along $\xi(t)$ and	
	the Acceleration of $\xi(t)$	
	19.F Stochastic Mechanics	107
20.	Mean Derivatives and Stochastic Mechanics	
	on Riemannian Manifolds	109
	20.A Mean Derivatives on Manifolds and	
	Related Equations	109
	20.B Geometric Stochastic Mechanics	114
	20.C The Existence of Solutions in Stochastic Mechanics	
21.	Relativistic Stochastic Mechanics	125
Part II	I. Infinite-Dimensional Differential Geometry and	
	Hydrodynamics	
<u> </u>		
Chapte	try of Manifolds of Diffeomorphisms	133
22.	Manifolds of Mappings and Groups of Diffeomorphisms	
	22.A Manifolds of Mappings	
	22.B The Group of $H^s$ -Diffeomorphisms	
	22.C Diffeomorphisms of a Manifold with Boundary	190
	22.D Some Smooth Operators and	195
	Vector Bundles over $D^s(M)$	194
23.	Weak Riemannian Metrics and Connections	190
	on Manifolds of Diffeomorphisms	108
	23.A The Case of a Closed Manifold	
	23.B The Case of a Manifold with Boundary	
	23.C The Strong Riemannian Metric	14
<b>24</b> .	Lagrangian Formalism of Hydrodynamics	1 40
	of an Ideal Barotropic Fluid	142
	24.A Diffuse Matter	
	24.B A Barotropic Fluid	143

Chapte		
Lagran	gian Formalism of Hydrodynamics of an Ideal	
Incomp	ressible Fluid	147
<b>25</b> .	Geometry of the Manifold of Volume-Preserving	
	Diffeomorphisms and LHSs of an Ideal Incompressible Fluid .	147
	25.A Volume-Preserving Diffeomorphisms	
	of a Closed Manifold	148
	25.B Volume-Preserving Diffeomorphisms	
	of a Manifold with Boundary	151
	25.C LHS's of an Ideal Incompressible Fluid	
26.		
	on a Manifold with Boundary as an LHS	
	with an Infinite-Dimensional Constraint	
	on the Group of Diffeomorphisms of a Closed Manifold	156
27.		100
21.	on the Existence of Solutions	164
	on the Existence of Boldwolls	104
Chapte		
	lynamics of a Viscous Incompressible Fluid and	
	stic Differential Geometry	
	aps of Diffeomorphisms	171
541 546		111
28.		1 50
	Diffeomorphisms of the <i>n</i> -Dimensional Torus	
29.	A Viscous Incompressible Fluid	175
		170
V-2	dices	
Α.	Introduction to the Theory of Connections	
	Connections on Principal Bundles	
	Connections on the Tangent Bundle	180
	Covariant Derivatives	181
	Connection Coefficients and Christoffel Symbols	183
	Second-Order Differential Equations and the Spray	185
	The Exponential Map and Normal Charts	186
В.	Introduction to the Theory of Set-Valued Maps	186
C.	Basic Definitions of Probability Theory and	
	the Theory of Stochastic Processes	188
	Stochastic Processes and Cylinder Sets	
	The Conditional Expectation	
	Markovian Processes	
	Martingales and Semimartingales	
D.	The Itô Group and the Principal Itô Bundle	
E.	Sobolev Spaces	
	Accessible Points and Closed Trajectories of	
	Mechanical Systems (by Viktor L. Ginzburg)	192
	Growth of the Force Field and Accessible Points	

Table of Conf	tents	XV	
Accessible Points in Systems with Constraints. Closed Trajectories of Mechanical Systems			
References		203	
Index		211	

· ·

e