

Contents

<i>Preface</i>	v
<i>References</i>	xiii

PART ONE. SYSTEMS WITH A FINITE NUMBER OF DEGREES OF FREEDOM

Chapter 1 Formulation of the Problem and Development of Notation	3
1.1 Introduction	3
1.2 Standardization of Notation	5
1.3 Matrices	7
1.4 Elementary Arithmetic Operations with Matrices	9
1.5 The Row-Column Rule	12
1.6 Warnings	13
1.7 Some Properties of Determinants	14
1.8 Inverses	16
1.9 Linear Independence	17
Chapter 2 Solution for Diagonalizable Matrices	21
2.1 Solution by Taylor Series	21
2.2 Eigenvalues and Eigencolumns	22
2.3 Superposition	23
2.4 Completeness	24
2.5 Diagonalization of Nondegenerate Matrices	27
2.6 Outline of Computation Procedure with Examples	29
2.7 Change of Variable	32
2.8 The Steady-state Solution	34
2.9 The Inhomogeneous Problem	35
Chapter 3 The Evaluation of a Function of a Matrix for an Arbitrary Matrix	38
3.1 Introduction	38
3.2 The Cauchy-integral Formula	38
3.3 Application to Matrices	39
3.4 Evaluation of $f(A)$ with Illustrations	40

3.5 The Inversion Formula	43
3.6 Laplace Transforms	44
3.7 Inhomogeneous Equations	46
3.8 The Convolution Theorem	47
Chapter 4 Vector Spaces and Linear Operators	50
4.1 Introduction	50
4.2 Base Vectors and Basis	52
4.3 Change of Basis	55
4.4 Linear Operators	57
4.5 The Representation of Linear Operators by Matrices	57
4.6 The Operator in the Dual Space	58
4.7 Effect of Change of Basis on the Representation of an Operator	59
4.8 The Spectral Representation of an Operator	60
4.9 The Formation of a Basis by Eigenvectors of a Linear Operator	61
4.10 The Diagonalization of Normal Matrices	64
Chapter 5 The Dirac Notation	67
5.1 Introduction	67
5.2 The Change of Basis	68
5.3 Linear Operators in the Dirac Notation	69
5.4 Eigenvectors and Eigenvalues	70
5.5 The Spectral Representation of an Operator	70
5.6 Theorems on Hermitian Operators	70
Chapter 6 Periodic Structures	73
6.1 Motivation	73
6.2 The <i>RC</i> Line	73
6.3 Diagonalizing M	75
6.4 The Loaded String	78
6.5 Difference Operators	79
PART TWO. SYSTEMS WITH AN INFINITE NUMBER OF DEGREES OF FREEDOM	
Chapter 7 The Transition to Continuous Systems	85
7.1 Introduction	85
7.2 The <i>RC</i> Line—Change of Notation	85
7.3 The <i>RC</i> Line—Transition to the Continuous Case	87
7.4 Solution of the Discrete Problem	87
7.5 Solution in the Limit (Continuous Problem)	89
7.6 The Fourier Transform	92
Chapter 8 Operators in Continuous Systems	96
8.1 Introduction	96
8.2 Operators on Functions	97
8.3 The Dirac δ Function	98
8.4 Coordinate Transformations	100
8.5 Adjoints	103
8.6 Orthogonality of Eigenfunctions	104

8.7 Functions of Operators	105
8.8 Three-dimensional Continuous Systems	107
8.9 Differential Operators	108
Chapter 9 The Laplacian (∇^2) in One Dimension	111
9.1 Introduction	111
9.2 The Infinite Domain, $-\infty < x < +\infty$	112
9.3 The Semi-infinite Domain, $0 \leq x < +\infty$	116
9.4 The Finite Domain, $0 \leq x \leq L$	119
9.5 The Circular Domain	122
9.6 The Method of Images	123
Chapter 10 The Laplacian (∇^2) in Two Dimensions	128
10.1 Introduction	128
10.2 Conduction of Heat in an Infinite Insulated Plate; Cartesian Coordinates	128
10.3 The Vibrating Rectangular Membrane	130
10.4 Conduction of Heat in an Infinite Insulated Plate; Plane Polar Coordinates	131
10.5 Theorems on Cylindrical Functions (of Integral Order n)	134
10.6 Conduction of Heat in an Infinite Insulated Plate; Plane Polar Coordinates (Concluded)	140
10.7 The Circular Membrane	141
10.8 The Vibrating Circular Ring and Circular Sector	143
Chapter 11 The Laplacian (∇^2) in Three Dimensions	146
11.1 Introduction	146
11.2 The Wave Equation in Three Dimensions	147
11.3 The Eigenvalues of L^2 and L_z	149
11.4 The Simultaneous Eigenfunctions of L^2 and L_z	152
11.5 Solution of $\nabla^2\psi = 0$	154
11.6 Solution of $(\nabla^2 + k^2)\psi = 0$	155
11.7 Recurrence Relations for the Spherical Harmonics	158
11.8 Some Expansion Theorems	160
11.9 Solution of the Wave Equation	162
11.10 Heat Conduction in an Infinite Solid	163
Chapter 12 Green's Functions	165
12.1 Definition	165
12.2 The Necessary and Sufficient Condition for a Green's Function	166
12.3 The Operator $-\alpha^2 d^2/dx^2 + 1$ in an Infinite Domain	167
12.4 The Operator $-\alpha^2 d^2/dx^2 + 1$ in a Finite Domain	169
12.5 The Operator $-\alpha^2 \nabla^2 + 1$ in Spherical Coordinates	171
Chapter 13 Radiation and Scattering Problems	176
13.1 The Outgoing Wave Condition	176
13.2 The Green's Function Solution	177
13.3 The Multipole Expansion	179
13.4 The Radiation Far from the Source	180
13.5 Radiation from an Infinitely Long Cylinder	181
13.6 The Scattering Problem	183

13.7	The Scattering Cross Section	184
13.8	The Method of Partial Waves	185
13.9	The Born Approximation	190
13.10	Gratings	192

PART THREE. APPROXIMATE METHODS

Chapter 14	Perturbation of Eigenvalues	201
14.1	Introduction	201
14.2	Formulation of the Problem	202
14.3	A Simple Solution	203
14.4	Nondegenerate Eigenvalues	204
14.5	Change of Notation and an Extension	205
14.6	An Application—The Vibrating String	206
14.7	Degenerate Eigenvalues	207
Chapter 15	Variational Estimates	211
15.1	Introduction	211
15.2	The Rayleigh Variational Principle	212
15.3	A Lower Bound	213
15.4	The Ritz Method	215
15.5	Higher Eigenvalues by the Ritz Method	220
15.6	Example of the Ritz Method	221
Chapter 16	Iteration Procedures	225
16.1	Introduction	225
16.2	Eigenvalue Problems	225
16.3	Inverses by Iteration	229
Chapter 17	Construction of Eigenvalue Problems	233
17.1	Introduction	233
17.2	The Method	233
17.3	Application to the Scattering Problem	234
Chapter 18	Numerical Procedures	236
18.1	Introduction	236
18.2	Simplification of the Model	236
18.3	Difference Equations from the Variational Principle	239
Appendix 1A	Determinants	243
Appendix 1B	Convergence of Matrix Power Series	252
Appendix 1C	Remarks on Theory of Functions of Complex Variables	254
1C.1	Analytic Functions	254
1C.2	The Cauchy Integral Theorem and Corollary	255
1C.3	Singularities	256
1C.4	Cauchy's Integral Formula	257
1C.5	The Theorem of Residues	259

Appendix 2A Evaluation of Integrals of the Form	$\int_{-\infty}^{+\infty} F(x)e^{izx} dx$	261
Appendix 2B Fourier Transforms, Integrals, and Series	266
2B.1 Introduction	266
2B.2 Transforms	267
2B.3 Infinite One-dimensional Transforms	268
2B.4 Infinite Multidimensional Transforms—Cartesian Coordinates	271
2B.5 Finite One-dimensional Transforms	272
2B.6 The Fourier-Bessel Integral	275
2B.7 The Fourier-Bessel Expansion	276
Appendix 2C The Cylindrical Functions	280
2C.1 Introduction	280
2C.2 The Integral Representation of $J_n(\rho)$	280
2C.3 The Integral Representations of the General Cylindrical Functions	281
2C.4 The Integral Representation of the Bessel Function J_ν	283
2C.5 The Hankel Functions	284
2C.6 Series Expansions at the Origin	286
2C.7 The Asymptotic Expansions	287
2C.8 The Asymptotic Series of Debye	291
2C.9 The Addition Theorems for Bessel Functions	292
Index	295