## **CONTENTS**

PREFA	ACE .	ix
1	PRELIMINARIES	1
		•
	1.1 The Difference Operators	1
	1.2 Theory of Interpolation	3
	1.3 Finite Difference Equations	10
	1.4 Linear Systems with Constant Coefficients	13
	1.5 Distribution of Roots of Polynomials	15
	1.6 First Integral Mean Value Theorem	18
	1.7 Common Norms in ODEs	18
2	NUMERICAL INTEGRATION ALGORITHMS	21
	2.1 Introduction	21
	2.2 Existence of Solution, Numerical Approach	25
	2.3 Special IVPs	23 27
	2.4 Error Propagation, Stability and	21
	Convergence of Discretization Methods	30
	Convergence of Discretization withouts	30
3	THEORY OF ONE-STEP METHODS	33
	3.1 General Theory of One-Step Methods	33
	3.2 The Euler Scheme, the Inverse Euler Scheme	
	and Richardson's Extrapolation	37
	3.3 The Convergence of Euler's Scheme	41
	3.4 The Trapezoidal Scheme	44
4	RUNGE-KUTTA PROCESSES	51
	4.1 General Theory of Runge-Kutta Processes	51
	4.2 The Explicit Two-Stage Process	59
	4.3 Convergence and Stability of Two-Stage	
	Explicit R-K Scheme	64
	4.4 Matrix Representation of the R-K Processes	66
	4.5 Error Estimation and Stepsize Selection	

vi CONTENTS

	in R-K Processes	76
	4.6 Implicit and Semi-Implicit R-K Processes	79
	4.7 Rosenbrock Methods	89
5	LINEAR MULTISTEP METHODS	93
	5.1 Starting Procedure	93
	5.2 Explicit Linear Multistep Methods	95
	5.3 Implicit Linear Multistep Methods	100
	5.4 Implementation of the Predictor-Corrector Formulas	105
	5.5 General Theory of Linear Multistep Methods	109
	5.6 Automatic Implementation of the Adams Scheme	127
6	NUMERICAL TREATMENT OF SINGULAR/DISCONTINUOUS	
	INITIAL VALUE PROBLEMS	131
	6.1 Introduction	131
	6.2 Non-Polynomial Methods	133
	6.3 The Inverse Polynomial Methods	140
	6.4 Local Error Estimates in Automatic	
	Codes for Discontinuous Systems	145
7	EXTRAPOLATION PROCESSES AND SINGULARITIES	147
	7.1 Introduction	147
	7.2 Generation of the Zero-th Column of	
	Extrapolation Table	149
	7.3 Polynomial and Rational Extrapolation	155
	7.4 Convergence and Stability Properties	1.00
	of Extrapolation Processes	163
	7.5 Practical Implementation of	1.65
	Extrapolation Processes	167
8	STIFF INITIAL VALUE PROBLEMS	169
	8.1 The Concept of Stiffness	169
	8.2 Stiff and Nonstiff Algorithms	175
	8.3 Solution of Nonlinear Equations	

Contents	VII
Contents	***

	and Estimation of Jacobians	177
	8.4 Region of Absolute Stability	181
	8.5 Stability Criteria for Stiff Methods	189
	8.6 Stronger Stability Properties of IRK Processes	198
	8.7 One-Leg Multistep Methods	206
9	STIFF ALGORITHMS	209
	9.1 What are Stiff Algorithms	209
	9.2 Efficient Implementation of Implicit	
	Runge-Kutta Methods	214
	9.3 The Backward Differentiation Formula	223
	9.4 Second Derivative Formulas	228
	9.5 Extrapolation Processes for Stiff Systems	230
	9.6 Mono-Implicit Runge-Kutta Methods	236
10	SECOND ORDER DIFFERENTIAL EQUATIONS	239
	10.1 Introduction	239
	10.2 Linear Multistep Methods and the	
	Concept of P-Stability	241
	10.3 Derivation of P-Stable Formulas	244
	10.4 One-Leg Multistep Methods for Second Order IVPs	245
	10.5 Multiderivative Methods	250
11	RECENT DEVELOPMENTS IN ODE SOLVERS	253

## **REFERENCES**