

CONTENTS

LIST OF CONTRIBUTORS	v
PREFACE	vii

Chapter 1. Mathematical Theory of Equilibrium Cracks

J. N. Goodier

I. Introduction	2
II. Character of Boundary Value Problems and Their Solutions. Linearly Elastic Continuum	5
III. The Illustrative Boundary Value Problem of Fig. 1 in Complete Plane Strain	8
IV. The Illustrative Boundary Value Problem of Fig. 1 as Simple Tension with Plane Deformation Hole	11
V. Essential Limitations Imposed by the Postulates of the Linearly Elastic Continuum	12
VI. The Boundary Value Problem of Fig. 3—Simple Tension with Plane Deformation Elliptic Hole	13
VII. Statement and Examination of Griffith's Theory	18
VIII. Barenblatt's Theory	22
IX. Connection of Barenblatt and Griffith Theories	30
X. Dissipation of Energy in Plastic Flow at Crack Tips	34
XI. A Hypothetical Criterion for Crack Extension in a Ductile Metal	40
XII. The Essential Nonlinearity of Extending Crack Problems	42
XIII. The Locally Nonlinear Theory of Goodier and Kanninen	43
XIV. Recommended Research	48
XV. Summary	50
Appendix A. The Representation of Plane Deformation States by Two Complex Potentials	51
Appendix B. The Linear Boundary Value Problem of Simple Tension with the Plane Deformation Elliptic Hole (see Sect. VI)	53
Appendix C. Energy Theorems Concerning the Formation and Extension of Cracks and Other Internal Free Boundaries in the Linearly Elastic Solid	56
Appendix D. Derivation of Eq. (53) from Eq. (51) (Sect. VIII)	59
Appendix E. Derivation of Eqs. (66), (67) (Sect. VIII) from Eqs. (46), (49) (Sect. VIII)	61
Appendix F. Derivation of Eq. (70) (Sect. IX)	62
Symbols	65
References	66

Chapter 2. Mathematical Theories of Brittle Fracture*G. C. Sih and H. Liebowitz*

I. Introduction	68
II. The Griffith Theory	72
III. Irwin's Theory of Fracture	89
IV. Rectilinearly Anisotropic Bodies with Cracks	108
V. Theory of Cracks in Three Dimensions	131
VI. Special Problems and Recommendations for Future Research	166
VII. Summary	185
Symbols	186
References	188

Chapter 3. Mathematical Analysis in the Mechanics of Fracture*James R. Rice*

I. Introduction	192
II. Preliminaries and Relevant Concepts from the Mechanics of Solids	193
III. Linear Elasticity in the Analysis of Deformation and Fracture	213
IV. Plasticity in the Analysis of Deformation and Fracture	243
V. Recommended Research	301
VI. Summary	304
Symbols	306
References	308

Chapter 4. Macroscopic Criteria for Plastic Flow and Brittle Fracture*B. Paul*

I. Introduction	315
Part One: Yielding of Ductile Metals	
II. Plastic Flow under Uniaxial Stress	316
III. Initial-Yield Criteria for Ductile Isotropic Metals under Multiaxial States of Stress	327
IV. Initial Yielding of Anisotropic Materials	348
V. Subsequent Yield Criteria and Constitutive Equations	354
VI. Experiments on Yielding of Metals under Biaxial States of Stress	364
VII. Additional Factors Which Influence Yielding under Multiaxial Stress	382
Part Two: Brittle Fracture under Multiaxial States of Stress	
VIII. General Comments on Brittle Fracture under Multiaxial States of Stress	385
IX. The Coulomb–Mohr Criterion of Failure	386
X. Generalized Pyramidal Failure Surfaces for Isotropic Materials	401
XI. Applications to Real Materials	413

Part Three: Relationship between Griffith's Theory and Macroscopic Fracture Criteria

XII.	Griffith's Criterion for Biaxial Stress States	426
XIII.	Initial and Subsequent Fracture Loci	429
XIV.	Implications of Subsequent Failure Loci for Compression Testing	440

Part Four: Closure

XV.	Historical Review	446
XVI.	Summary, Conclusions, and Recommended Research	453
	Appendix A. Definitions and Notation for Stress Components	455
	Appendix B. Transformation of Plane States of Stress, Mohr's Circle	459
	Appendix C. Transformation of Three-Dimensional States of Stress	462
	Appendix D. Existence and Determination of Principal Stresses	465
	Appendix E. Mohr's Circles for Three-Dimensional States of Stress	468
	Appendix F. Some Properties of the Stress Deviation Tensor	473
	Appendix G. Root Mean Square Shear Stress	479
	Appendix H. Pyramidal Parameters as a Function of Strength Parameters	481
	Appendix I. Derivation and Generalization of Griffith's Fracture Criterion for Biaxial States of Stress	482
	Symbols	487
	References	489

Chapter 5. **Crack-Propagation Theories**

Fazil Erdogan

I.	Introduction	498
II.	Dynamic Crack-Propagation Theories	502
III.	Fatigue Crack Propagation	561
	Appendix A. An Estimate of the Plastic Zone in the Shear Problem	583
	Appendix B. Evaluation of Various Energies in the Plastic Region	584
	Symbols	585
	References	586

Chapter 6. **Statistical Approach to Brittle Fracture**

Alfred M. Freudenthal

I.	Introduction	592
II.	Statistical Concepts in Fracture Mechanics	593
III.	Applications	610
IV.	Recommended Research	616
V.	Summary	617
	Symbols	618
	References	618

Chapter 7. Theory of Micropolar Elasticity

A. Cemal Eringen

I. Introduction	622
II. Deformation and Microdeformation	626
III. Strain and Microstrain Tensors	631
IV. Micropolar Strains and Rotations	635
V. Geometrical Meaning of Micropolar Strains and Rotations	640
VI. Invariants of Strain Tensors	646
VII. Volume Changes	648
VIII. Compatibility Conditions	650
IX. Some Special Deformations	652
X. Motion, Micromotion, and the Material Derivative of Tensors	659
XI. Velocity, Acceleration, Microrotation, and Spin	661
XII. Material Derivative of Arc Length	665
XIII. Rates of Strain Measures	669
XIV. External and Internal Loads	670
XV. Mechanical Balance Laws	674
XVI. Stress and Couple Stress	679
XVII. Local Balance Laws	683
XVIII. Conservation of Energy	686
XIX. Principle of Entropy	688
XX. Theory of Micropolar Elasticity	690
XXI. Restrictions on Micropolar Elastic Moduli	693
XXII. Field Equations, Boundary and Initial Conditions	695
XXIII. Indeterminate Couple Stress Theory	698
XXIV. Propagation of Waves in an Infinite Micropolar Elastic Solid	702
XXV. Reflection of a Longitudinal Displacement Wave	707
XXVI. Micropolar Surface Waves	711
XXVII. Stress Concentration around a Circular Hole	713
XXVIII. Galerkin and Papkovitch Representation	719
XXIX. A Micropolar Infinite Solid Subjected to a Concentrated Force and a Concentrated Couple	722
XXX. Recommended Research	725
XXXI. Summary	725
Symbols	726
References	728
 AUTHOR INDEX	 730
SUBJECT INDEX	736