## CONTENTS

	002(222	-1-0				
LIST	OF PLATES	•	•		•	x
	VOLUM	E I I				
	PART 2. THE ANALYSIS A		_	SOF		
	GEOMAGNET	IC DAT	.'A			
Intro	DDUCTION TO PART 2	•	•	•	•	543
XVI.	PERIODICITIES AND HARMONIC AND	ALYSIS 1	N GEOPH	YSICS		545
1.	Periodic and non-periodic functions					545
	The non-cyclic variation					546
	The choice of days					546
	The effect of curvature					547
5.	Elimination of the non-periodic part					548
	Sine-waves: Fourier series					549
	Orthogonality					551
	Approximation to a function by a fir	nite trige	onometrice	ıl series		552
	Approximation to a function by a functions				onal	553
10.	Bessel's inequality	•	•	•	•	554
	Orthogonal polynomials. Legendre for	unctions	•	•	•	554
	Harmonic analysis of a set of equidis			inction	•	556
	The connexion between harmonic and				•	558
	The relation between the Fourier series				and	000
	the series obtained from equidistar			·	and .	559
15.	Corrections for non-cyclic variations		•			560
	Gibbs's phenomenon and the converg	gence of	the Fourie	er series		561
	The influence of smoothing .					562
18.	The harmonic dial		•			563
19.	Numerical harmonic analysis .		•	•	566,	605
	Mechanical and optical harmonic ana	lysers	•	•		569
	Graphical harmonic analysis: physica	•	ries .			569
	The theory of errors		•	•		572
23.	The asymptotic solution for the rand	om walk		•		575
	The random walk with unit displacer					578
	The harmonic components of a rando		f numbers			579
	Effects of observational errors on har					581
27.	Random series, and series with conse	rvation	•			582
	Conservation in magnetic character-fi		unspots. a	nd morta	lity	
	data	•	•	•		585
29.	Statistical test for periodicities; the p	persisten	ce curve			<b>586</b>
30.	Persistence in relation to the periodo	gram .	•	•		589
	The summetion-dial					593

32.	The generalized harmonic dial .	•	•	•		<b>593</b>
33.	Example: The 27-day recurrence pheno	omenon	•			594
	Example: The annual variation of mag	•	•	•	•	601
35.	Example: The semi-annual persistent	wave in	magnet	ic chara	cter-	
0.0	figures	•	•	•	•	601
36.	Concluding remarks	•	•	•	•	603
XVII	I. SPHERICAL HARMONIC ANALYSIS IN	<b>GEOPHY</b>	SICS	•		606
	The representation of arbitrary function			a spher	e .	606
	Zonal harmonics					607
3.	Associated functions					609
4.	Normalized spherical functions .			•		610
	Developments in series of spherical har	monics				612
	The functions $P_n^m$ expressed in terms of		ots.			613
7.	Tesseral and sectorial surface harmonic	s .				615
	Spherical harmonic functions as solution		olace's e	quation		615
	Development for the reciprocal radius			•		616
	The general spherical surface harmonic	function	S <sub>n</sub>			617
	Expression of $S_n$ in terms of the function			•		619
	Transformation and other formulae		•			620
13.	Separation of a field of force, known of	ver a sp	herical	surface.	into	
	parts of external or internal origin	. 1				624
	Application to terrestrial magnetism	•	•	•		626
	An integral theorem on spherical harmo	onics	•	•	•	627
	A theorem of potential theory .	•	•	•	•	628
	The potential of a magnetic double-layer			•	•	629
18.	The magnetic potential of a steady cu	ırrent-di	stributio	on in a	thin	
10	spherical shell	•	•	•	•	630
	Numerical calculation of the spherical h			ents	•	631
	The mutual dependence of the spherical	l harmon	ics	•	•	633
	Schuster's method of development.	•	•	•	•	634
	Schmidt's and other methods .	•	•	•	•	637
23.	Concluding remarks	•	•	•	•	637
	I. THE SPHERICAL HARMONIC ANALYS	SIS OF TH	ie Main	7 FIELD		639
	Gauss's analysis and its repetitions.	•	•	•		639
	The analysis by Schmidt		•	•	•	640
	The first-order harmonic	•	•	•	•	642
	Geomagnetic coordinates and time	•	•		•	645
	The field of the centred dipole .		•	•	•	646
	The magnetic centre C and the eccentric	c dipole	•	•	•	648
	The terms of higher order	•	•	•		659
	The analysis by Dyson and Furner	•	•	•	•	663
	The external field, and the non-potentia		•	•	•	663
	The decrease of the field of internal original	gin, abov	e the ea	rth	•	665
	The secular variation	•	•	•	•	666
12.	Multipoles			•		668

XIX. THE VARIABILITY OF			COEFF	TCIENT	S FOR TH	łΕ
Solar and Lunar Daily V	'ARIA	TIONS		•	•	. 66
1. Methods for expressing th	ıe var	iability of d	laily va	riations		. 66
2. Elliptical clouds of points	3	•		•	•	. 67
3. The daily variation of de-	clinat	ion at Huai	ncavo i	n summ	er	. 67
4. The daily variation of ho			•		•	. 67
5. Simultaneous changes of S		•			at the san	
station		•	•	•	•	. 67
6. Quasi-persistence in S on	succe	ssive days		•	•	. 67
7. S and $\overline{L}$ in Huancayo $H$		•	•	•	•	67
8. S and L on individual day	s (for	H at Huar	ncayo)		•	. 68
XX. THE SPHERICAL HARI			_	MILE	MAGNET	TO
VARIATIONS	MONIC	2 Analys	IS OF	THE	MAGNET	. 68
1. Introduction .	•	•	•	•	•	. 68
	•	4:-1 - C - J	:1		•	. 68
2. General expression for the	_		-	riation	•	
3. The assumption of no dep	<b>ena</b> ei	nce on long	ituae	•	•	. 68
4. Chapman's analysis of S	. 1	•	•	•	•	. 68
5. Comparison with other ar	-		•	•	•	. 69
6. Separation of internal and	d exte	ernal terms	•	•	•	. 69
7. Analyses of the L field	•	•	•	•	•	. 69
8. The similarity of S and L	•	•	•	•	•	. 69
9. McNish's analysis .	•	•	•	•	•	. 69
10. Hasegawa's analyses	•	•	•	•	•	. 69
11. Analysis of the $D_{st}$ field	•	•	•	•	•	. 69
D a DIIVOTAL MILEO	DIEC	OF CEO	N.F. A. CO.N.T.	TOTAL	DILENTO	A TONT A
PART 3. PHYSICAL THEO	KIES	OF GEO	MAGN	ETIC	PHENO	
Introduction to Part 3	•	•	•	•	•	. 69
XXI. THEORIES OF THE MAIN	Firi	D AND ITS	SECUL	AR VAF	RIATION	. 70
1. The problem of the field						. 70
2. The assumption of perma					•	. 70
3. Internal currents .		11106110012001		•	•	. 70
4. Gyromagnetic effects	•	•	•	•	•	. 70
5. Rotating electric charges	•	•	•	•	•	. 70
6. The bearing of the secula		ation on the	· oorios o	· ·f tha m	oin field	. 70
3						. 70
7. The magnetic effects of east. The external field .	21 (11-1)	novements	and oce	an cum	CIIUS	
	•	•	•	•	•	. 71
XXII. ELECTROMAGNETIC INI	OUCTI	ON WITHIN	THE E	ARTH	•	. 71
1. Induction by the external	l part	of the S, L,	and D	fields	•	. 71
2. The uniform-core model of	ompa	tible with t	he S an	nd L field	ds	. 71
3. The influence of the surfa	ce lay	vers .	•	•	•	. 71
4. The distribution of the in	duced	l currents ir	the co	ore	•	. 72
5. The magnetic permeabilit	y wit	hin the eart	h	•	•	. 72
6. Induction by aperiodic fie	_	•	•			. 72
7. Induction by the storm-ti		isturbance f	$\mathbf{P}_{st}$			. 72
8. Non-uniform core models					l $D_{st}$ fields	3. 72
9. The distribution of condu	_				•	. 73

THE MATHEMATICAL THEORY OF CURRENT-INDUCTION			
NETIC FIELDS IN REGIONS BOUNDED BY CONCENT	RIC	SPHERES	. 732
10. The general equations	•	•	. 732
11. Solutions for regions bounded by spheres .	•	•	. 735
12. Non-conducting regions	•	•	. 735
13. Conducting regions	•	•	. 736
14. The conditions satisfied at a spherical boundary	•	•	. 737
15. The functions $f_n(r)$	•	•	. 737
16. Induction in a uniform solid sphere .	•	•	. 740
17. The 'free' current-systems		•	. 741
18. Lamb's second solution for A		•	. 742
19. Periodic induction in a uniform solid sphere	•	•	. 742
20. Aperiodic induction in a uniform solid sphere	•	•	. 744
21. Plane-earth induction problems	•	•	. 749
XXIII. THEORIES OF THE SOLAR AND LUNAR ]	DAIL	Y MAGNET	TC
Variations $S_q$ and $L$	•	•	. 750
A. THE DYNAMO THEORY OF $S$ AND $L$ .		•	. 750
1. The external part of the $S_q$ and L fields .			. 750
2. Untenable hypotheses			. 750
3. Balfour Stewart's dynamo theory			. 751
4. Schuster's development of the dynamo theory			. 752
5. Later investigations of the dynamo theory of S ar	nd L		. 755
6. The phase-law in L			. 756
7. The dynamo theory of the L field			. 757
8. Comparison with radio data			. 758
9. Daily and seasonal changes of the ionospheric cor	nduci	tivity	. 760
10. The differences between the S and L fields.			. 761
11. A qualitative derivation of the L current-circuit from	m th	ne tidal moti	
12. Outline of the mathematical development of the			. 768
13. The intensification of S and L at Huancayo			. 776
B. THE DIAMAGNETIC AND DRIFT-CURRENT THEORIES	OT 6	•	
14. The diamagnetic theory	OF. 2	•	. 780
15. The drift-current theory	•	•	. 780
16. Criticisms of both theories	•	•	. 782
17. A fundamental error in the simple drift-current th	•	•	. 783
	eory	•	. 787
18. The inadequacy of the diamagnetic theory of \$	•	•	. 788
C. ATMOSPHERIC IONIZATION AND THE DAILY MAGNET			. 790
19. Conclusions from the radio and magnetic effects of	of sol	ar flares	. 790
20. The effect of the sunspot-cycle on $S$ and $L$ .	•	•	. 792
21. The day-to-day changes in S		•	. 793
22. The effect of a solar eclipse on the S current-syste	m	•	. 794
XXIV. CORPUSCULAR EMISSIONS FROM THE SUN, AN	an C	TEOM A CINITERE	ro
DISTURBANCE	ט עי	· MOMAGNET!	. 799
1. Corpuscular streams, or bursts of ultra-violet light	t <b>?</b>	•	. 799

CONTENTS	ix
----------	----

2.	The emission of corpuscular streams	•	•		801
3.	The geometry of corpuscular streams		•		804
4.	The influence of the heliographic latitude of	f the so	urce		808
	The penetration of the corpuscles into the			ere .	809
	The detection of the corpuscular streams b				810
	The composition of the solar stream		•		813
8.	Solar eclipses, and ionization by neutral co	rpuscles	· ·		814
	The electrical state of corpuscular streams			a the sun	
	The magnetic field produced by the moving				828
	General account of the Birkeland-Störmer				833
	Motion of a single charged particle in the g				834
	Störmer's equatorial current-ring .				842
	The action of a magnetic field on a neutral	ionized	stream		845
	0			•	0.10
	. Theories of Magnetic Storms and A	URORAE	}		850
	The principal facts to be explained	•			850
	The nature of the solar cause .	•			850
	The advance of a neutral ionized stream cearth's magnetic field	•	•		853
4.	The distortion of the advancing stream-sur of a magnetic storm	rface, ar	nd the fi	rst phase	855
5.	The quantitative discussion of the first pha	se			859
	The main phase of a magnetic storm	•			866
	Aurorae and other features of magnetic sto	rms			869
	The ultra-violet light theory of aurorae and		tic storn	ns .	870
	The location of the inter-zonal D currents				876
10.	The polar part of the D current-system	•			881
	The irregular features of magnetic disturba	nce, D.			887
	Magnetic potential energy and field energy	, _ •			888
	The energy of superposed magnetic fields				890
	The field-energy changes during magnetic s		•		892
	50 5 6 6			•	002
<b>37 37 37</b>	T TY.				
	I. HISTORICAL NOTES		•	•	898
	The directive property of the magnet in the				898
	The discovery of magnetic declination: reje		early cl	aims .	901
	The actual discovery of magnetic declination	n	•		904
	The earliest observations of declination	•	•		907
	The discovery of magnetic inclination	•	•		909
	The discovery of the secular variation				910
	The earliest magnetic charts .				911
	William Gilbert	•	•		914
	The discovery of non-secular magnetic time				922
10.	The connexion between magnetic storms are	nd polar	lights .		923
	The hypothesis of four magnetic poles				925
12.	C. F. Gauss	•	•		927
13.	Magnetic observatories and observations				929

x CONTENTS

14. The Göttingen Magne	etic Uni	ion	•	•			931
15. E. Sabine and the Br	ritish Co	lonial c	bservato	ories			933
16. The introduction of p	hotogra	aphic re	cording	•	•	•	938
17. Summary of importar to about 1850	it event	s in the	history	of geom	nagnetisi	n up	936
BIBLIOGRAPHY .	•		•	•			938
INDEX OF AUTHORS	•	•				•	1009
INDEX OF SUBJECTS	•		•		•	•	1018
L	IST	OF P	LAT	ES			
	VO	LUM	E $II$				
35. Birkeland's and Brüche	's terrel	la expe	riments	•	•	•	. 831
36. Störmer's diagrams of e	xcluded	l spaces	in the t	heory of	f the au	ora	. 837
37. Examples (stereograms)	of corp	uscular	paths ca	lculated	l by Stö	rmer	. 842
38. Halley's Atlantic chart.	of the r	naoneti	c 'variat	tion'	Retween	nages (	916_11