CONTENTS

PART I

I. FORMS AND DESCRIPTIONS

1. Forms described in the photographic atlas 1
2. Description of some auroral displays 5

II. GEOGRAPHICAL DISTRIBUTION OF AURORAE. THE NORTHERN AND SOUTHERN AURORA BELTS. THE MOTION OF AURORA DURING THE NIGHT AND DURING MAGNETIC STORMS. PERIODS OF AURORA

3. Geographical distribution of aurora. Aurora belts 14
4. The motion of the aurora towards the tropics during great magnetic storms 17
5. The simultaneous occurrence of Northern and Southern Lights 18
6. The connexion between great aurorae and magnetic storms. Early researches by Celsius, Hiorter, and Wilcke. Bartels's diagrams of the magnetic three-hour indices K 18
7. The periods and variation with time of the aurora 25

III. METHODS OF OBSERVING AND PHOTOGRAPHING AURORAE

8. Schemes for visual observation 33
9. The problem of photographing the aurora 36

IV. VISUAL AND PHOTOGRAPHIC METHODS OF DETERMINING HEIGHT AND POSITION OF THE AURORA

10. Early attempts to determine height and position of the aurora by visual observations 40
11. The photographic method of finding height and position of an aurora. Amount of material collected 41

V. THE METHOD OF MEASUREMENT OF THE PLATES. METHODS USED TO SIMPLIFY THE WORK

12. Historical remarks 45
13. Formulae for H and D 46
14. Formulae from spherical trigonometry to find the angles necessary for calculating H and D 48
15. Maurer-Becker nomograms for solving spherical triangles 51
16. Remarks on the practical use of Becker's nomograms for aurora researches 54
17. Practical methods of finding t, g, δ_0, and t_0 using a calculating machine 55
18. Construction of nets for use in determining the coordinates of the selected aurora points 56
19. Graphical methods for the determination of r, H, and D 59
20. Continuation of the projection method described in section 12 62
21. The case of sunlit aurorae 65

VI. RESULTS OF THE MEASUREMENTS OF AURORA PLATES FROM REGIONS NEAR THE NORTHERN AURORA BELT.

22. Results from the author's expeditions to Bossekop in 1910 and 1913 67
23. The results of the height measurements by Krogness and Vegard at Bossekop (1912–13) and Haldde (1913–14) 72
24. Studies of aurorae made by Harang and Tönsberg from the aurora observatory at Tromsö in the year 1929–30 and later 74
25. On an auroral arc with deep crimson lower border penetrating to between 60 and 70 km over the earth, observed by Harang and Bauer on 8 March 1932 80
26. Work by Krogness and Tönsberg with base line Bossekop–Haldde of 12·35 km. Height and motion of forms DS and PS 80

VII. MEASUREMENTS OF AURORAE NOT SITUATED NEAR THE AURORA BELTS

27. Measurements of aurorae made by the French–Norwegian Micardbu expedition to north-eastern Greenland, in collaboration with the Danish expedition 1938–9 to the same region 81
28. Preliminary results from Thule during the polar year 1932–3 83
29. Observations and photographic measurements of aurorae from New Zealand made by the late M. Geddes 83

VIII. RESULTS OF OBSERVATIONS AND PHOTOGRAPHIC WORK IN SOUTHERN NORWAY FROM 1911–52

30. Material collected 89
31. Distribution of the heights of all the measured auroral points from 1911–44 91
32. Height and geographical position of homogeneous auroral arcs HA 92
33. Height and geographical position of bands with ray structure RB 104
34. Height and geographical position of cloudlike aurora DS 107
35. Auroral rays in the dark atmosphere and some height statistics of them 111
36. Observations and measurements of other auroral forms 111
37. Isolated pulsating arcs 111
38. Pulsating spots. Their periods 114
39. Split and rapidly pulsating arcs 118
40. The remarkable cloudlike aurora on 3 January 1940 119
41. Remarkable auroral arc of short isolated rays photographed by Westin from Trondheim on 24 March 1936 122
42. Remarkable quiet auroral bands, RB, with ray structure on 7–8 January 1937 123
43. Exceptionally high auroral rays on 22–23 March 1920 126
44. On a remarkable grey-violet auroral curtain with rays in sunlight observed on 8 September 1926 126

IX. GENERAL REMARKS ON SUNLIT AURORA

45. The position of sunlit rays relative to the earth's shadow line. Frequency and colour 128
46. The divided auroral rays 132
47. Sunlit auroral arcs 132

X. THE INTENSITY AND COLOUR OF THE AURORA AND ASSOCIATED PHENOMENA. MOVING AND COLOUR PICTURES OF THE AURORA

48. The intensity of the aurora 135
49. A case of aurora seen in full daylight 135
50. Peculiar sounds associated with the most intense aurora 137
51. Remarks on the numerous observations of intense aurorae penetrating to lower levels than the photographic measurements have hitherto shown 139
52. Moving and colour pictures 140

XI. THE COLOUR OF THE AURORA

53. The colour of the different auroral forms 142
54. Remarkably rapid changes in the colour of rays from blue-green to red all over 143

XII. THE VISUAL AND PHOTOGRAPHIC MEASUREMENTS OF THE AURORAL SPECTRUM

55. Historical remarks 145
56. Typical auroral spectra 146
57. The common yellow-green aurora line 5577 Å 147
58. The red oxygen lines 6300, 6364, and 6391 Å 147
59. The hydrogen lines H_α, H_β, and H_γ 148
60. The Doppler effect of the hydrogen lines and their shift towards the violet, found by Vegard 150
61. The great shift towards the violet of the hydrogen line H_α found by Meinel during the great aurora 18–20 August 1950 152
62. Supplementary researches by Gartlein and Vegard 154
63. The other lines in the auroral spectrum 158
64. Vegard's spectrograms taken in Oslo 23–24 February 1950 and in Tromsö during the winter 1950–1 158
65. Recent results from spectrographic researches by Meinel at Yerkes Observatory in U.S.A. and by Petrie at Saskatoon in Canada 160
66. Variations of the auroral spectrum with auroral form, height, latitude, and other conditions 161
67. The mechanism of emission of the auroral light 163
68. The method used by Vegard to determine the temperature in the auroral region 163
69. Some results obtained by photographic height measurements combined with simultaneous photographs of auroral spectra 164
70. Collaboration between Professor Berkey and Norwegian aurora specialists to obtain simultaneous height measurements and spectra with a grating spectrograph of high dispersion 166

XIII. SPECTRA OF SUNLIT AURORAL RAYS

71. Spectra of sunlit auroral rays obtained in 1929, 1936, and 1938. Their possible polarization 167

XIV. RADIO WAVES OF 10-CM WAVELENGTH EMITTED BY THE AURORA

72. The observations by Forsyth, Petrie, and Currie from Saskatoon in 1949 171

XV. POSSIBLE MEASUREMENTS OF CORPUSCULAR CURRENTS OUTSIDE THE EARTH'S ATMOSPHERE

73. Interest of experiments 173
74. Wireless echoes of long delay 173
75. Further observations of echoes 177
76. Various attempts to give a theoretical explanation of the wireless echoes of long delay 178
77. The position of the sun when echoes were heard 180
78. Recent attempts to obtain echoes of long delay 181
79. Some suggestions for new researches on wireless echoes of long delay 182

XVI. WIRELESS ECHOES FROM THE AURORA

80. Introduction 183
81. Echoes observed at the Jodrell Bank experimental station near Manchester 183
82. Echoes observed from Saskatoon, Canada 184
83. Echoes observed from Kiruna, northern Sweden 185
84. Echoes observed from Tromsö and Kjeller, near Oslo, in the winter 1952–3 with remarks regarding their origin 186

XVII. EVIDENCE OF STREAMS OF CORPUSCLES BETWEEN THE SUN AND THE EARTH BY DOPPLER EFFECT OF ABSORPTION LINES IN THE SOLAR SPECTRUM

85. Researches by Richardson, Brück, and Ruttlant 188

XVIII. ABSORPTION OF CORPUSCULAR RAYS IN THE ATMOSPHERE

86. Introductory remarks 190
87. Information necessary for solving the problems of the penetration and luminosity of corpuscular rays when they produce what we call the aurora 190
88. Formula for the pressure as a function of height. Scale height 191
89. Lenard's absorption formula for cathode rays penetrating from space down into the atmosphere 192
90. The author's researches of 1912 194
91. A method of determining the temperature from the length of auroral rays 194
92. Further researches in 1921 by Vegard and the author 195
93. Vegard's latest results regarding the absorption of electrons and protons in the upper atmosphere 198
94. Harang's researches on what he calls the luminosity curve of the aurora, which furnish a method of finding the scale height 199
95. Some results of the recent experiments with V2 and other rockets reaching heights of some hundred kilometres 200
96. Concluding remarks 203

XIX. EMISSION OF ELECTRIC CORPUSCLES FROM THE SUN

97. Introduction 205
98. Solar eruptions, radio fade-outs, and subsequent aurorae. Carrington's discovery 205
99. Aurorae not related to solar flares. *M*-regions 207

PART II

I. FUNDAMENTAL PROBLEM. DIFFERENTIAL EQUATIONS

1. The first ideas before Birkeland's experiments 209
2. Birkeland's experiments 210
3. Experiments by Villard, Brüche, and Malmfors 212
4. Mathematical problems to solve. Simplifying assumptions 212
5. The differential equations of motion of an electric corpuscle in a magnetic field 214
6. The equations of motion in the case of a magnetic dipole. Reduction by means of a new unit of length 216
7. A second 'first integral' of the equations of motion 218
8. The problem of integration 219
9. Other forms for the differential equations useful for the discussion and the numerical calculation of the trajectories 223

II. RESULTS OBTAINED BY DIRECT STUDY OF THE DIFFERENTIAL EQUATIONS

10. Geometrical interpretation of the equation for $d\varphi/ds$. Allowed and forbidden regions 229
11. Discussion of the curves $[\gamma, \kappa]$ and of the allowed regions q_γ and Q_γ 231
12. Some general remarks on the trajectories defined by the equations (6.4) 239
13. Some general properties of the trajectories 239
14. Method of discussion using the coordinates R, z, and φ 242

III. SPECIAL FAMILIES OF ORBITS

15. Orbits and trajectories passing through the origin 247
16. Other families of orbits in the Rz-plane 248

IV. NUMERICAL INTEGRATIONS

17. The method used. Numerical integration of systems of differential equations of the second order 255
18. Application of the results of numerical integration to the trajectories 260
19. Example of numerical integration using the system (9.15) 261

V. INTUITIVE METHOD OF CONTROL

20. Geometrical-mechanical method for checking and discussing the orbits in the $R_1 z_1$-plane 264

VI. SOME RESULTS OF THE NUMERICAL INTEGRATIONS

21. Orbits in the Rz-plane passing through the origin discussed in the field $U =$ constant 269
22. The corresponding trajectories in space going to the dipole 278
23. Trajectories in the vicinity of the trajectories through the origin (dipole) 279
24. Alfvén's perturbation method 284

VII. APPLICATION OF THE MATHEMATICAL THEORY TO PHYSICAL EXPERIMENTS

25. Introduction of the 'stiffness' $H\rho$ of the corpuscle against magnetic action 288
26. Application to some of Birkeland's experiments 289
27. Application to some experiments of Villard, Brüche, and Malmfors 290

VIII. APPLICATION TO THE POLAR AURORA

28. Introduction. Values of $H\rho$, v, V, and the unit C_{st} 292
29. Regions of precipitation of auroral corpuscles. Southern limit of aurora borealis 294
30. Limit of the auroral regions on the side of the geomagnetic poles. The belts of maximum frequency of the aurora 297
31. Explanation of the occurrence of the aurora during the night. The case of a bundle of trajectories corresponding to the same value of the constant γ 299
32. Distinguished directions. Importance of the study of the trajectories through the origin in explaining the forms and development of aurorae 302
33. The rapid variations of the aurora. Occurrence of the same auroral form on two consecutive nights almost at the same hour 306
34. Possible explanation of the fact that the aurora has a tendency to recur after 27 days 307
35. Explanation of long and narrow arcs with ray structure 307
36. Numerical example of the formation of a long and narrow arc 313
37. Explanation of the horseshoe-like auroral curtains 319
38. The application of the method to non-homogeneous bundles of trajectories to explain auroral forms 323
39. Explanation of the quiet homogeneous arcs HA 323
40. The transition of a homogeneous arc HA into an arc with ray structure RA 325
41. Explanation of auroral rays 327
42. Explanation of auroral draperies 331
43. Explanation of other auroral forms 332

IX. WHAT THE MATHEMATICAL THEORY CAN OR CANNOT EXPLAIN

44. Auroral phenomena which may be explained by the corpuscular theory in its original form and others which may not be so explained 335

X. FURTHER DEVELOPMENT OF THE CORPUSCULAR THEORY ACHIEVED BY DISCARDING THE SIMPLIFYING ASSUMPTIONS ONE AFTER THE OTHER SO AS TO APPROACH THE ACTUAL CONDITIONS IN NATURE

45. Introductory remarks 338
46. The action on a single corpuscle of neighbouring corpuscles when they are sent out from the sun simultaneously as a cloud or a stream 338
47. Vegard's hypothesis of a stream where the velocities of positive and negative corpuscles are different 339
48. Action of an external ring current on the situation of the aurora belts 340
49. Equations of motion in the case where the magnetic field is symmetric round the z-axis 340
50. Some useful properties of the function ϕ 343
51. The case of a single current in the xy-plane 344
52. Results of the discussion 345
53. On the trajectories in the magnetic dipole field when the corpuscles move under the additional action of a central force inversely proportional to the square of the distance, and emanating from the dipole 346
54. Equations of motion and reductions 346
55. U-fields and allowed and forbidden regions 348

56. The case of solar magnetism 358
57. The magnetic field of the earth according to the complete Gaussian series 359
58. The remaining dropped assumptions 360

XI. OTHER AURORA THEORIES
59. The theory of Chapman and Ferraro 362
60. The theory of Martyn 363
61. The theory of Alfvén 367
62. Further theories of aurora 372

XII. OTHER APPLICATIONS OF THE MATHEMATICAL RESEARCHES INTO THE MOTION OF ELECTRIC CORPUSCLES IN THE FIELD OF A MAGNETIC DIPOLE
63. Introductory remarks 374
64. Application to cosmic rays. Introduction 374
65. Allowed and forbidden regions according to the mathematical theory 376
66. The forms of the differential equations used during the detailed studies of the trajectories 376
67. Short summary of the work by Lemaître, Vallarta, and their pupils 378
68. The author's work on the theory of cosmic rays coming from an infinite distance 381
69. Application to the streams in the solar corona during minima of solar activity and method of finding the general magnetic field of the sun 384
70. Application to the Eschenhagen oscillations of the earth's magnetism 387

XIII. CONCLUDING REMARKS
71. References to topics not included in this book 388

LIST OF NAMES 393

LIST OF SUBJECTS 396

PAPERS AND NEWS ON AURORA 400

FIGURES 1–16, 29–34, 83, 85, 91, 95–96, 101–4, 113–16, 123–4, 126, 142, 149–51, 154, 156–64, 174, 184–5, 191–3, 206–7, and 212–13 appear as separate plates at the end of the book.