

Contents

Contents of Volumes I and II	xi
Introduction	xiii
13 Function Space and Operator Theory for Nonlinear Analysis	1
Introduction	1
1 L^p -Sobolev spaces	2
2 Sobolev imbedding theorems	3
3 Gagliardo-Nirenberg-Moser estimates	7
4 Trudinger's inequalities	12
5 Singular integral operators on L^p	15
6 The spaces $H^{s,p}$	22
7 L^p -spectral theory of the Laplace operator	28
8 Hölder spaces and Zygmund spaces	37
9 Pseudodifferential operators with nonregular symbols	45
10 Paradifferential operators	54
11 Young measures and fuzzy functions	66
12 Hardy spaces	77
References	85
14 Nonlinear Elliptic Equations	89
Introduction	89
1 A class of semilinear equations	91
2 Surfaces with negative curvature	101
3 Local solvability of nonlinear elliptic equations	108
4 Elliptic regularity I (interior estimates)	115
5 Isometric imbedding of Riemannian manifolds	125
6 Minimal surfaces	130
6B Second variation of area	145
7 The minimal surface equation	152
8 Elliptic regularity II (boundary estimates)	159
9 Elliptic regularity III (DeGiorgi-Nash-Moser theory)	169
10 The Dirichlet problem for quasi-linear elliptic equations	180

11	Direct methods in the calculus of variations	192
12	Quasi-linear elliptic systems	198
12B	Further results on quasi-linear systems	211
13	Elliptic regularity IV (Krylov-Safonov estimates)	223
14	Regularity for a class of completely nonlinear equations	236
15	Monge-Ampere equations	243
16	Elliptic equations in two variables	254
A	Morrey spaces	258
B	Leray-Schauder fixed-point theorems	261
	References	263
15	Nonlinear Parabolic Equations	271
	Introduction	271
1	Semilinear parabolic equations	272
2	Applications to harmonic maps	282
3	Semilinear equations on regions with boundary	288
4	Reaction-diffusion equations	290
5	A nonlinear Trotter product formula	307
6	The Stefan problem	315
7	Quasi-linear parabolic equations I	327
8	Quasi-linear parabolic equations II (sharper estimates)	336
9	Quasi-linear parabolic equations III (Nash-Moser estimates)	344
	References	354
16	Nonlinear Hyperbolic Equations	359
	Introduction	359
1	Quasi-linear, symmetric hyperbolic systems	360
2	Symmetrizable hyperbolic systems	370
3	Second-order and higher-order hyperbolic systems	376
4	Equations in the complex domain and the Cauchy-Kowalewsky theorem	387
5	Compressible fluid motion	390
6	Weak solutions to scalar conservation laws; the viscosity method	398
7	Systems of conservation laws in one space variable; Riemann problems	412
8	Entropy-flux pairs and Riemann invariants	436
9	Global weak solutions of some 2×2 systems	445
10	Vibrating strings revisited	453
	References	459
17	Euler and Navier-Stokes Equations for Incompressible Fluids	466
	Introduction	466

1	Euler's equations for ideal incompressible fluid flow	467
2	Existence of solutions to the Euler equations	476
3	Euler flows on bounded regions	485
4	Navier-Stokes equations	493
5	Viscous flows on bounded regions	505
A	Regularity for the Stokes system on bounded domains	515
	References	520
18	Einstein's Equations	524
	Introduction	524
1	The gravitational field equations	525
2	Spherically symmetric spacetimes and the Schwarzschild solution	533
3	Stationary and static spacetimes	545
4	Orbits in Schwarzschild spacetime	554
5	Coupled Maxwell-Einstein equations	560
6	Relativistic fluids	563
7	Gravitational collapse	572
8	The initial-value problem	578
9	Geometry of initial surfaces	587
10	Time slices and their evolution	597
	References	602
	Index	607