# CONTENTS


PREFACE . . . . . xiii
ACKNOWLEDGMENTS . . . . . xv
GENERAL REFERENCES . . . . . xvii

PART I

**COHERENT NONLINEAR PHENOMENA**

Chapter 1. **Introduction**

1.1 Basic Equations and Methods . . . . . 3
1.2 Terminology and Classification of Interactions . . . . . 6
1.3 Multiple-Time-Scale Perturbation Analysis of the Van der Pol Equation . . 8
References . . . . . 13

Chapter 2. **The Korteweg–de Vries Equation—A Weakly Nonlinear Theory of Ion Sound Waves**

2.1 Introduction and Basic Equation . . . . . 15
2.2 Derivation of the Korteweg–de Vries Equation for Nonlinear Ion Sound Waves . . . . . 19
2.2.1 Ion Sound Wave Model . . . . . 19
2.2.2 Solitary Wave Solution . . . . . 20
2.2.3 The Korteweg–de Vries Equation for Nonlinear Ion Sound Waves . 22
2.3 Solution of the Korteweg–de Vries Equation . . . . . 23
2.4 Examples and General Properties of the Solution . . . . . 27
2.5 Nonlinear Theory of Ion Sound Waves with Collisionless Dissipation . . . 29
References . . . . . 31
Supplementary References . . . . . 32

Chapter 3. **Large-Amplitude Electron Plasma Oscillations**

3.1 One-Dimensional Cold-Plasma Model . . . . . 33
3.2 Exact Solution in Lagrangian Variables . . . . . 35
3.3 Nonlinear Example with Inversion to Eulerian Variables . . . . . 38

3.4 Extensions of the Model . . . . . 41
3.4.1 Large-Amplitude Oscillations at the Upper Hybrid Frequency . . . 41
3.4.2 Modifications Due to Collisional Drag . . . . . 43
3.5 Modifications Due to Finite Electron Temperature . . . . . 44
3.5.1 The Truncated Moment Description . . . . . 44
3.5.2 The Single-Water-Bag Model . . . . . 45
3.5.3 Analytic Properties of the Nonlinear System . . . . . 47
References . . . . . 52
Supplementary References . . . . . 53

Chapter 4. **Strong Wave–Particle Interactions—Particle Trapping**

4.1 Introduction . . . . . 54
4.2 Collisionless Damping of Electron Plasma Oscillations . . . . . 57
4.2.1 Review of Linear Theory . . . . . 57
4.2.2 Breakdown of Linearization . . . . . 62
4.2.3 Effects of Particle Trapping on the Damping of a Monochromatic Wave . . . . . 63
4.2.4 Localized Electric Field Disturbances . . . . . 70
4.3 Large-Amplitude Bernstein–Greene–Kruskal Waves . . . . . 72
4.3.1 Equilibrium Theory . . . . . 72
4.3.2 Example of a Large-Amplitude BGK Wave . . . . . 76
4.4 Stability of Bernstein–Greene–Kruskal Waves . . . . . 80
References . . . . . 87
Supplementary References . . . . . 87

Chapter 5. **Plasma Echoes**

5.1 Introduction and Basic Echo Mechanism . . . . . 89
5.2 Theory of Second-Order Temporal Echoes . . . . . 90
References . . . . . 98
Supplementary References . . . . . 99

Chapter 6. **Coherent Nonlinear Wave–Wave Interactions**

6.1 Introduction and Basic Equation . . . . . 101
6.2 Wave–Wave Coupling in Macroscopic Plasma Models . . . . . 103
6.3 Wave–Wave Coupling in Vlasov Plasma Models . . . . . 107
6.4 Nonlinear Theory of Resonant Three-Wave Coupling . . . . . 116
6.4.1 Basic Equation . . . . . 116
6.4.2 Conservation Laws . . . . . 117
6.4.3 Nonlinear Stability Theorem . . . . . 118
6.4.4 Exact Solution for a Single Resonant Triplet . . . . . 119
6.5 Examples of Resonant Three-Wave Coupling . . . . . 124
References . . . . . 127
Supplementary References . . . . . 128



PART II

## TURBULENT NONLINEAR PHENOMENA

Chapter 7. **Statistical Frameworks for Weak Plasma Turbulence**

7.1 Basic Definitions and Terminology . . . . . 133
7.2 The Random Phase Approximation . . . . . 135
7.3 Evolution of Collective Correlations in a Vlasov Ensemble . . . . . 137
7.3.1 The Vlasov Cumulant Hierarchy . . . . . 137
7.3.2 Range of Validity of the Random Phase Approximation . . . . . 141
7.3.3 Relation between the BBGKY and Vlasov Cumulant Hierarchies . . 143
7.3.4 Weak Turbulence Ordering of the Vlasov Cumulant Hierarchy . . . 143
7.4 Discrete Particle Interactions . . . . . 146
7.5 Summary . . . . . 147
References . . . . . 148
Supplementary References . . . . . 149

Chapter 8. **Weak Turbulence Theory of Electrostatic Wave–Particle Interactions**

8.1 Introduction and Basic Assumptions . . . . . 151
8.2 The Weak Turbulence Kinetic Theory . . . . . 152
8.2.1 Truncated Vlasov Cumulant Hierarchy . . . . . 152
8.2.2 Derivation of the Kinetic Equations . . . . . 154
8.3 Weak Turbulence Kinetic Equations for a Multicomponent Plasma . . . . 160
8.4 General Properties of the Kinetic Equations . . . . . 161
8.4.1 Conservation Laws . . . . . 161
8.4.2 The Velocity-Space Diffusion Tensor . . . . . 163
8.4.3 Extension of the Theory to the Stable Regime . . . . . 164
8.5 Electrostatic Turbulence in a Magnetoplasma . . . . . 165
8.6 Phase-Mixing of Free-Streaming Correlations . . . . . 166
References . . . . . 172
Supplementary References . . . . . 173

Chapter 9. **Stabilization of the Weak Electron Beam (Bump-in-Tail) Instability**

9.1 Introduction and Dispersion Relation . . . . . 174
9.2 Stabilization in One Dimension . . . . . 175
9.3 Stabilization in Higher Dimensions . . . . . 182
9.3.1 General Considerations . . . . . 182
9.3.2 The Two-Dimensional Ring . . . . . 183
9.4 Range of Validity of the Theory . . . . . 187

9.5 Computer Simulation of the Weak Bump-in-Tail Instability . . . . . 189
9.6 Resonance Broadening . . . . . 193
References . . . . . 196
Supplementary References . . . . . 197

## Chapter 10. Stabilization of the Ion Loss-Cone Instability

10.1 Introduction and Basic Assumptions . . . . . 198
10.2 Dispersion Relation . . . . . 200
10.3 Stabilization Process . . . . . 201
References . . . . . 205

## Chapter 11. Electron Heating by Electron–Ion Beam Instabilities

11.1 Introduction . . . . . 206
11.2 Dispersion Relation . . . . . 207
11.2.1 General Considerations . . . . . 207
11.2.2 Symmetric Counterstreaming Ion Beams . . . . . 209
11.3 Time Evolution of the Electron–Ion Beam System . . . . . 212
11.3.1 General Considerations . . . . . 212
11.3.2 Symmetric Counterstreaming Ion Beams . . . . . 215
11.4 Computer Simulation of the Electron–Ion Beam Instability . . . . . 217
References . . . . . 220
Supplementary References . . . . . 220

## Chapter 12. Weak Turbulence Theory of Electromagnetic Wave–Particle Interactions

12.1 The Vlasov–Maxwell Description . . . . . 222
12.2 Weak Turbulence Kinetic Theory for Parallel Propagation . . . . . 225
12.2.1 Basic Assumptions and Solution for the Perturbed Distribution Function . . . . . 225
12.2.2 Adiabatic Dispersion Relation . . . . . 228
12.2.3 Weak Turbulence Kinetic Equations . . . . . 229
12.3 Stabilization of the Electron Whistler Instability . . . . . 231
12.4 Stabilization of the Garden-Hose Instability . . . . . 234
References . . . . . 241
Supplementary References . . . . . 242

## Chapter 13. Weak Turbulence Theory of Nonlinear Wave–Wave Interactions

13.1 Introduction . . . . . 243
13.2 The Hierarchy for Wave Correlations . . . . . 245



13.3 The Weak Turbulence Kinetic Theory . . . . . 248
13.3.1 Weak Turbulence Ordering and Multiple-Time-Scale Perturbation Expansion . . . . . 248
13.3.2 Kinetic Equation for Three-Wave Processes . . . . . 249
13.3.3 Resonant Three-Wave Kinetic Equation for Electrostatic Plasma Waves . . . . . 252
13.4 General Properties of the Wave Kinetic Equation . . . . . 253
13.4.1 Preservation of the Nonnegative Nature of $N^{\alpha}_{\mathbf{k}_1}$ . . . . . 253
13.4.2 Conservation Laws . . . . . 254
13.4.3 Theorem for Nonlinear Stability . . . . . 255
13.4.4 Nonequilibrium Entropy . . . . . 255
13.5 Exact Solution of the Wave Kinetic Equation in One Dimension . . . . . 258
13.6 Kinetics of Resonant Three-Wave Interactions in Three Dimensions . . . 262
13.6.1 Diffusion of Electron Plasma Oscillations in a Large-Amplitude Ion Wave Background . . . . . 262
13.6.2 Solution of the Diffusion Equation . . . . . 264
13.7 Kinetic Equation for Resonant Four-Wave Processes . . . . . 267
References . . . . . 271
Supplementary References . . . . . 272

## Chapter 14. General Weak Turbulence Theory of Nonlinear Electrostatic Interactions

14.1 Introduction and Basic Assumptions . . . . . 274
14.2 Derivation of the Kinetic Equations . . . . . 274
14.2.1 The Vlasov Cumulant Hierarchy . . . . . 274
14.2.2 Weak Turbulence Ordering and Multiple-Time-Scale Perturbation Expansion . . . . . 276
14.2.3 Order-$\epsilon$ Solution . . . . . 278
14.2.4 Order-$\epsilon^2$ Solution . . . . . 281
14.2.5 Kinetic Equation for the Waves . . . . . 285
14.2.6 Kinetic Equation for the Particles . . . . . 287
14.3 Extension of the Kinetic Equations to the Multicomponent Many-Mode Case 289
14.3.1 Some Generalizations and Useful Approximations . . . . . 289
14.3.2 Kinetic Equation for the Particles . . . . . 291
14.3.3 Kinetic Equation for the Waves . . . . . 293
14.4 Properties of the Kinetic Equations . . . . . 294
14.4.1 Conservation Laws . . . . . 294
14.4.2 Limiting Forms of the Kinetic Equations . . . . . 296
14.5 Nonlinear Landau Damping of Electron Plasma Oscillations . . . . . 298
14.5.1 The Kinetic Equations in One Dimension . . . . . 298
14.5.2 Evolution of the Density of Electron Plasma Oscillations . . . . . 303
14.5.3 Computer Simulation of Nonlinear Landau Damping . . . . . 307
14.6 Extensions of the Weak Turbulence Theory . . . . . 308
14.6.1 Discrete Particle Interactions . . . . . 308
14.6.2 Weak Spatial Nonuniformities . . . . . 314
References . . . . . 315
Supplementary References . . . . . 316

Appendix A. **The Korteweg–de Vries Equation and the Inverse Scattering Problem** . . . . . 318

Appendix B. **Matrix Symmetries** . . . . . 321

Appendix C. **Phase-Mixing of Free-Streaming Correlations** . . . 323

Appendix D. **Second-Order Two-Body Correlations in a Weakly Turbulent Plasma**

D.1 Evaluation of the Two-Body Correlation Function . . . . . 324
D.2 Kinetic Equation for the Waves . . . . . 332
D.3 Kinetic Equation for the Particles . . . . . 333
Reference . . . . . 333

Appendix E. **Nonlinear Wave–Particle Coupling Coefficients**

E.1 Coupling Coefficient for the Wave Kinetic Equation . . . . . 334
E.2 Coupling Coefficient for the Particle Kinetic Equation . . . . . 337
Reference . . . . . 339

Author Index . . . . . 341
Subject Index . . . . . 348